AD FALCON API Manual

Uncoupled Dynamic Analysis with FALCON

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1 Uncoupled Dynamic Analysis with FALCON

1.1 Introduction

This reference summarises the governing equations for uncoupled solid mechanics—mass, momentum, and energy balances without fluid coupling. In this formulation, pore pressure effects are either neglected or incorporated through prescribed total stresses, and no fluid flow equations are solved.

All symbols used below are listed once in **Table 1**.

Implementation note (FALCON): In `*NonCoupled` / `*UnCoupled` analyses, FALCON enforces `rho = rhos`, where `rhos` is read from `@PhaseChar: Solid rhos <...>`. Void ratio and saturation are not used to update density in uncoupled analyses.

1.2 Table 1: Variable Definitions

Symbol	Description
$M^s,\ \Omega^s$	mass, volume of solid phase
$\rho,\ \rho_s$	continuum density used in inertia/body forces, solid grain density (often $\rho = \rho_s$ in uncoupled FALCON input)
n	porosity
$\boldsymbol{\sigma}$	total stress tensor
$\mathbf{u},\ \dot{\mathbf{u}},\ \ddot{\mathbf{u}}$	displacement, velocity, acceleration
ε	strain tensor
$\mathbf{b}$	body-force vector (e.g., gravitational acceleration)
$E_I,\ E_k,\ \mathcal{P}$	internal energy, kinetic energy, power
$\mathbf{N},\ \mathbf{B},\ \mathbf{L}$	shape, strain–displacement, differential operators
$\mathbf{M},\ \mathbf{K},\ \mathbf{C}$	global mass, stiffness, damping matrices
$\mathbf{D}$	constitutive (tangent) matrix
c	damping coefficient
$\mathbf{n}^*,\ \mathbf{I}_\sigma^\top$	outward unit normal, stress-traction operator
$\bar{\mathbf{t}}$	prescribed traction (Neumann BC)
$\mathbf{U}$	global displacement DOF vector
$\mathbf{F}$	global load vector
$\Delta t,\ \beta,\ \gamma$	time increment, Newmark parameters

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1.3 Conservation of Mass

For the solid phase only ($\alpha = s$):

$$\frac{\partial(\rho_s(1-n))}{\partial t} + \nabla \cdot \left(\rho_s(1-n)\dot{\mathbf{u}}\right) = 0 \tag{1}$$

Assumptions leading to Eq. (2): Solid grains are assumed incompressible with spatially uniform density ($\nabla\rho_s = 0$). The material time derivative $D(\cdot)/Dt = \partial(\cdot)/\partial t + \dot{\mathbf{u}} \cdot \nabla(\cdot)$ is applied to the solid-phase mass balance:

$$\frac{Dn}{Dt} = (1-n)\nabla \cdot \dot{\mathbf{u}} \tag{2}$$

1.4 Linear Momentum Balance

For the solid continuum, the linear momentum balance is

$$\nabla \cdot \boldsymbol{\sigma} + \rho\,\mathbf{b} = \rho\,\ddot{\mathbf{u}} \tag{3}$$

where:
- $\boldsymbol{\sigma}$ is the total stress tensor (effective stress in dry or drained conditions) - ρ is the density used in inertia and body-force terms (in `*NonCoupled` / `*UnCoupled` FALCON analyses, `rho` is taken directly from `@PhaseChar: Solid rhos ...`)
- $\mathbf{b}$ is the body-force vector (typically gravitational acceleration) - $\ddot{\mathbf{u}}$ is the solid acceleration

Note: In uncoupled analysis, there is no distinction between total stress and effective stress if pore pressures are not explicitly modeled.

1.5 Balance of Energy

$$\frac{DE_I}{Dt} = \mathcal{P} - \frac{DE_k}{Dt} \tag{4}$$

$$\frac{DE_k}{Dt} = \int_\Omega \rho\,\ddot{\mathbf{u}} \cdot \dot{\mathbf{u}}\,d\Omega \tag{5}$$

$$\mathcal{P} = \int_\Gamma \mathbf{t} \cdot \dot{\mathbf{u}}\,d\Gamma + \int_\Omega \rho\,\mathbf{b} \cdot \dot{\mathbf{u}}\,d\Omega \tag{6}$$

1.5.1 Internal-Energy Rate

For the solid continuum, the internal energy rate per unit volume is

$$\frac{De_I}{Dt} = \boldsymbol{\sigma} : \nabla\dot{\mathbf{u}} \tag{7}$$

Constitutive relation:

$$\dot{\boldsymbol{\sigma}} = \mathbf{D}\,\dot{\varepsilon} \tag{8}$$

where $\mathbf{D}$ is the elastoplastic tangent stiffness matrix.

1.6 Weak Form

1.6.1 Solid momentum balance

$$\int_{\Omega} \delta \mathbf{u}^{\top}(\mathbf{L}^{\top}\boldsymbol{\sigma} + \rho \mathbf{b} - \rho \ddot{\mathbf{u}})\, d\Omega + \int_{\Gamma_t} \delta \mathbf{u}^{\top}(\bar{\mathbf{t}} - \mathbf{I}_{\sigma}^{\top}\boldsymbol{\sigma})\, d\Gamma = 0 \tag{9}$$

where:

- $\delta \mathbf{u}$ is the virtual displacement
- $\mathbf{L}$ is the differential operator relating strain to displacement
- $\bar{\mathbf{t}}$ is the prescribed traction on Γ_t
- $\mathbf{I}_{\sigma}$ is the stress-traction operator

1.7 FE Discrete Equation

Unknown vector: $\mathbf{U}$ (displacement).

Kinematic framework: Small-strain formulation is used for the stiffness matrix unless geometric nonlinearity (updated Lagrangian formulation) is explicitly enabled for large-deformation problems.

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F} \tag{10}$$

where:

- $\mathbf{M}$ is the **mass matrix**
- $\mathbf{C}$ is the **damping matrix**
- $\mathbf{K}$ is the **stiffness matrix**
- $\mathbf{U}$ is the **global displacement vector**
- $\mathbf{F}$ is the **global force vector**

Note on the damping matrix $\mathbf{C}$:
In uncoupled dynamic analyses, $\mathbf{C}$ represents material damping, typically proportional to density and a damping coefficient.

(See Appendix A for detailed matrix definitions.)

1.7.1 Special Case: Static Analysis

For **static analysis**, all acceleration and velocity terms vanish ($\ddot{\mathbf{U}} = \mathbf{0},\ \dot{\mathbf{U}} = \mathbf{0}$), and Eq. (10) reduces to:

$$\mathbf{K}\mathbf{U} = \mathbf{F} \tag{10S}$$

This represents the classical static equilibrium problem where only the stiffness matrix and load vector are required. The solution provides the displacement field under static loading without any time-dependent effects.

1.8 Appendix A: Finite-Element Matrices and Vectors

1.8.1 Mass matrix

$$\mathbf{M} = \int_{\Omega} \mathbf{N}_u^{\top} \rho \, \mathbf{N}_u \, d\Omega \tag{11}$$

where: - $\mathbf{N}_u$ is the displacement shape function matrix - ρ is the material density

1.8.2 Damping matrix

$$\mathbf{C} = \int_{\Omega} \mathbf{N}_u^{\top} c \, \mathbf{N}_u \, d\Omega \tag{12}$$

where c is the damping coefficient.

1.8.3 Stiffness matrix

$$\mathbf{K} = \int_{\Omega} \mathbf{B}^{\top} \mathbf{D} \, \mathbf{B} \, d\Omega \tag{13}$$

where: - $\mathbf{B}$ is the strain-displacement matrix - $\mathbf{D}$ is the constitutive (tangent stiffness) matrix

1.8.4 Load vector

$$\mathbf{F} = \int_{\Omega} \mathbf{N}_u^{\top} \rho \, \mathbf{b} \, d\Omega + \int_{\Gamma_t} \mathbf{N}_u^{\top} \bar{\mathbf{t}} \, d\Gamma \tag{14}$$

where:
- $\mathbf{b}$ is the body-force vector (e.g., gravitational acceleration) - $\bar{\mathbf{t}}$ is the prescribed traction on the Neumann boundary Γ_t

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1.9 Time Integration

1.9.1 Newmark Method

For time discretization, the **Newmark method** is applied, following the classical formulation of Newmark (1959) and standard finite-element expositions (e.g., Bathe 1996; Hughes 2000).

Displacement update:

$$\mathbf{U}_{n+1} = \mathbf{U}_n + \Delta t\,\dot{\mathbf{U}}_n + \frac{\Delta t^2}{2}\left[(1-2\beta)\ddot{\mathbf{U}}_n + 2\beta\ddot{\mathbf{U}}_{n+1}\right] \tag{15}$$

Velocity update:

$$\dot{\mathbf{U}}_{n+1} = \dot{\mathbf{U}}_n + \Delta t\left[(1-\gamma)\ddot{\mathbf{U}}_n + \gamma\ddot{\mathbf{U}}_{n+1}\right] \tag{16}$$

Newmark coefficients:

$$a_0 = \frac{1}{\beta\Delta t^2}, \quad a_1 = \frac{\gamma}{\beta\Delta t}, \quad a_2 = \frac{1}{\beta\Delta t} \tag{17}$$

$$a_3 = \frac{1}{2\beta} - 1, \quad a_4 = \frac{\gamma}{\beta} - 1, \quad a_5 = \Delta t\left(\frac{\gamma}{2\beta} - 1\right) \tag{18}$$

Unconditional stability conditions:

$$\gamma \geq 0.5, \quad \beta \geq 0.25(0.5+\gamma)^2 \tag{19}$$

Common choices: - **Average acceleration (implicit):** $\gamma = 0.5,\ \beta = 0.25$ (unconditionally stable, no numerical damping) - **Linear acceleration:** $\gamma = 0.5,\ \beta = 1/6$ (conditionally stable) - **Generalized-α / HHT-α method:** Modified Newmark with effective parameters
$\gamma_{\text{eff}} = 0.5 + \alpha_f - \alpha_m,\ \beta_{\text{eff}} = 0.25\big(1 + \alpha_f - \alpha_m\big)^2$,
where α_m, α_f are the algorithmic damping parameters (FALCON uses `alpham`, `alphaf`; setting $\alpha_m = 0,\ \alpha_f = \alpha$ recovers the classical HHT-α variant).

> **Note**
>
> In FALCON, the default choice is $\alpha_m = 0,\ \alpha_f = 0$, which reduces the scheme to standard Newmark average acceleration (no additional algorithmic damping).

1.10 References

Newmark, N.M. (1959). *A method of computation for structural dynamics. Journal of the Engineering Mechanics Division*, ASCE 85(3), 67–94.

Bathe, K.J. (1996). *Finite Element Procedures*. Prentice Hall.

Hughes, T.J.R. (2000). *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. Dover Publications.