Hysteretic Soil Water Retention Curve (SWRC) Model

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Contents

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1.1 Syntax

Specify this model in `% Materials` using `@SWRC::`:

```
@SWRC: Hysteretic alpha_1 <alpha_1> n <n> m <m> omega_prime <omega_prime>
alpha_2 <alpha_2> bd <bd> bw <bw> SW_max <SW_max> SW_min <SW_min> b_s_c
<b_s_c>
```

1.1.1 Parameters

Input keyword	Symbol	Meaning
`alpha_1`	α_1	Inverse air-entry of the main drying curve (at void ratio $e = 1$).
`alpha_2`	α_2	Inverse air-entry of the main wetting curve (at void ratio $e = 1$).
`n`	n	Shape parameter controlling slope.
`m`	m	Shape parameter controlling slope.
`omega_prime`	Ω'	Void-ratio influence in modified suction $p_c^* = p_c e^{\Omega'}$.
`bd`	b_d	Derivative correction exponent (drying branch).
`bw`	b_w	Derivative correction exponent (wetting branch).
`SW_max`	$S_{W_{max}}$	Maximum degree of saturation.
`SW_min`	$S_{W_{min}}$	Minimum degree of saturation.
`b_s_c`	b_{s_c}	Slope correction and smoothing exponent.

1.2   Introduction

The hysteretic SWRC model describes the relationship between soil suction, void ratio, and saturation. It employs a substepping integration scheme to accurately capture drying and wetting processes. This document presents the theoretical formulation, numerical integration method, and an example simulation.

1.3   Theory

The model employs a modified suction function that accounts for both suction and void ratio influences:

$$p_c^* = p_c e^{\Omega'} \tag{1}$$

where: - p_c is the suction, - e is the void ratio, and - Ω' is a material parameter controlling the void ratio effect on suction.

The rate of change of the effective saturation degree, S_e, is given by:

$$dS_e = M^* \, dp_c^* \tag{2}$$

with:

$$dp_c^* = \frac{\partial p_c^*}{\partial p_c} \, dp_c + \frac{\partial p_c^*}{\partial e} \, de \tag{3}$$

The model interpolates between the main wetting and drying curves using:

$$M^* = \left(\frac{p_c^*}{p_c^{*\alpha}} \right)^{b_\alpha} \frac{\partial S_e^\alpha}{\partial p_c^*} \tag{4}$$

where $\alpha = w, d$ for wetting and drying, respectively. The effective saturation evolution is computed as:

$$S_e^{(t+\Delta t)} = S_e^t + \int_0^{\Delta p_c^*} M^* \, dp_c^* \tag{5}$$

This equation is numerically integrated using an adaptive substepping scheme.

1.4   Numerical Integration

1.4.1   Substepping Scheme

1. **Initialize**: Set $T = 0$ and $DT = 1$.
2. **Compute Derivatives**: Evaluate derivatives at the current state.
3. **Predictor Step**: Compute trial effective saturation:

$$S_e^1 = S_e^t + \Delta S_e^1 \tag{6}$$

4. **Corrector Step**: Recompute derivatives and adjust saturation:

$$S_e^2 = S_e^t + 0.5(\Delta S_e^1 + \Delta S_e^2) \tag{7}$$

5. **Error Estimation**: Compute relative error and adjust step size DT.
6. **Update State**: If error is acceptable, update S_e and proceed.

1.5   Example Simulation

1.5.1   Material Properties

The following material parameters are used:

The following are the material and model parameters, with their values in the cming simulation.

Parameter	Description
SW_min	Minimum saturation
SW_max	Maximum saturation
alpha_1	Parameter (inverse of air-entry of the main drying curve at void ratio of 1.0) for drying branch
alpha_2	Parameter (inverse of air-entry of the main wetting curve at void ratio of 1.0) for wetting branch
n	Exponent parameter (drying branch)
m	Exponent parameter (drying branch)
omegaprime	Exponent for void ratio influence
alpha_p_c	Determines the position with respect to the main drying and wetting (1.0 is drying and 0.0 is wetting)
bd	Exponent for derivative correction (drying branch)
bw	Exponent for derivative correction (wetting branch)
b_s_c	Exponent for slope correction and smoothing

```
SW_min: 0.0
SW_max: 1.0
alpha_1: 0.05
alpha_2: 0.1
n: 2.0
m: 0.5
```

```
omegaprime: 2.0
alpha_p_c: 1.0
bd: 5.0
bw: 5.0
b_s_c: 25.0
```

1.5.2 Analysis Stages

The simulation consists of four stages:

```
# Stage 1 (Initial state defined)
stage: 1
suction: 10.0
initialSuction: 50.0
voidRatio: 0.4
suctionChange: 0.0
voidRatioChange: -0.2
numSteps: 1000

# Stage 2 (Incremental changes only)
stage: 2
suctionChange: 0.0
voidRatioChange: 0.3
numSteps: 1000

# Stage 3 (Incremental changes only)
stage: 3
suctionChange: 0.0
voidRatioChange: -0.2
numSteps: 1000

# Stage 4 (Incremental changes only)
stage: 4
suctionChange: 0.0
voidRatioChange: 0.3
numSteps: 1000
```

1.5.3 Results

The figure below shows the saturation evolution during the simulation.

The simulation successfully tracks the hysteretic behavior of the soil-water retention curve.

1.6 Conclusion

This document presents the formulation, numerical integration, and an example case for the hysteretic SWRC model. The method accurately captures the complex behavior of soil water retention under varying void ratios.
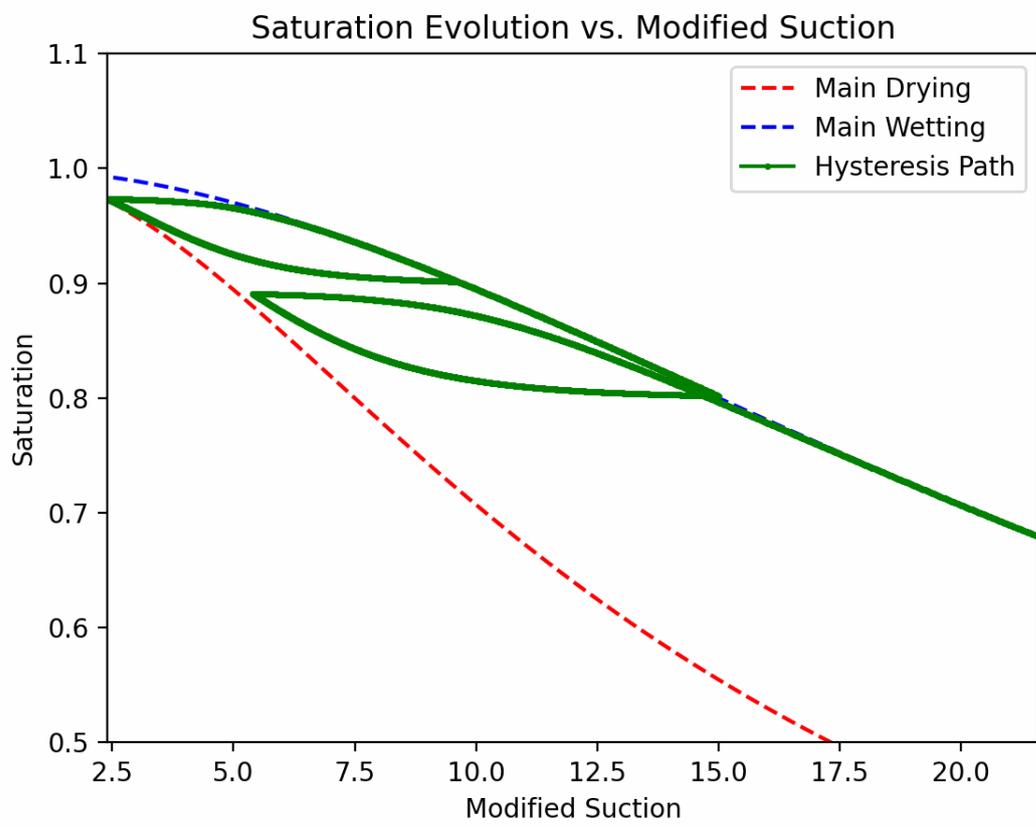
Figure 1: SWRC Results

The main drying and wetting branches use van-Genuchten-type retention curves, while the hysteretic scanning-curve framework and void-ratio coupling follow FALCON's unsaturated-soil implementation for deformable media.

1.7 References

- van Genuchten, M. Th. (1980). *A closed-form equation for predicting the hydraulic conductivity of unsaturated soils.* Soil Science Society of America Journal, 44(5), 892-898. https://doi.org/10.2136/sssaj1980.03615995004400050002x
- Ghorbani, J., Airey, D. W., & El-Zein, A. (2018). *Numerical framework for considering the dependency of SWCCs on volume changes and their hysteretic responses in modelling elasto-plastic response of unsaturated soils.* Computer Methods in Applied Mechanics and Engineering, 336, 80-110.