



AD FALCON API Manual

# Theory of the Sub/Super-Loading Surface Model (SSLSM)

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March 14, 2026

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# 1 Theory of the Sub/Super-Loading Surface Model (SSLSM)

This note summarizes the **Sub/Super-Loading Surface Model (SSLSM)** implementation used by the accompanying UMAT, following Zhao et al. (2005), *Explicit stress integration of complex soil models*.

SSLSM extends Modified Cam-Clay with:

- a **subloading surface**, so plastic straining can occur smoothly inside the conventional yield locus, and
- a **superloading surface**, used to represent initial structure and its degradation.

## 1.1 Syntax

This model is configured in % Materials as a user-defined mechanical material. Use @UMAT: with category Mechanical and pass parameters as name=value pairs.

Example:

```
@UMAT: path/to/SSLSMModel.cpp path/to/SSLSMModel.hpp Mechanical \
  Phi=35.38 Lambda=0.05 Kappa=0.035 Nu=0.3 UseLambdaStar=0 \
  Mmax=1.43 v_N=2.17 m_sub=0.127 a_sub=0.092 Ad_sub=0.95 \
  P_min=1.0 STOL=1e-7 FTOL=1e-6 LTOL=1e-6 \
  CustomVariable=IsotropicHardening,R,RS
```

For readability the example is wrapped across multiple lines; in input files the full @UMAT: directive should be written on a single line.

Example inputs used on this page:

- OCR triaxial validation: [sslsm\\_triaxial\\_ocr\\_sensitivity.txt](#)
- FEM strip-footing setup: [footing\\_strip\\_sslsm.txt](#)
- Footing runner: [run\\_sslsm\\_footing.py](#)

## 1.2 Material Parameters

Symbol	Keyword in input	Required	Meaning
$\phi$	Phi	✓	Critical-state friction angle.

Symbol	Keyword in input	Required	Meaning
$\lambda$	Lambda	✓	Virgin compression index. If UseLambdaStar=1, interpreted as $\lambda^*$ and converted using $v_N$ .
$\kappa$	Kappa	✓	Swelling/reloading index.
$\nu$	Nu	✓	Poisson ratio.
$v_N$	v_N	✓	Specific volume at $p = 1$ on the normal compression line.
$M_{\max}$	Mmax	✓	Maximum CSL slope used in Zhao et al. (2005) Eq. (1).
$m$	m_sub	✓	Evolution rate of the subloading ratio $R$ .
$a_{\text{sub}}$	a_sub	✓	Evolution rate of the structure ratio $R_n$ .
$A_d$	Ad_sub	✓	Weight used in the hardening measure $d\varepsilon_d$ .
$P_{\min}$	P_min	✓	Minimum pressure used in elastic stiffness evaluation.
–	STOL	✓	Substepping tolerance.
–	FTOL	✓	Yield tolerance.
–	LTOL	✓	Load-unload tolerance.

Following Zhao et al. (2005), the critical-state slope varies with the Lode angle:

$$M(\theta) = M_{\max} \left[ \frac{2a_{\text{Lode}}^4}{1 + a_{\text{Lode}}^4 - (1 - a_{\text{Lode}}^4) \sin(3\theta)} \right]^{1/4}, \quad a_{\text{Lode}} = \frac{3 - \sin \phi}{3 + \sin \phi} \quad (1)$$

### 1.3 Custom State Variables

Declare custom variables using CustomVariable= in the @UMAT: line.

Name	Required	Meaning
Isotropic Hardening	✓	$p_c$ : size of the normal Cam-Clay surface.
R	✓	Subloading-to-super similarity ratio.
RS	✓	Normal-to-super similarity ratio; this stores paper's $R_n$ .
OCR	optional	OCR used during initialization.
InitOnSubloading	optional	If 1, force the initial isotropic state onto the subloading surface.
UpdateVoidRatio	optional	If 1, rebuild the initial void ratio to match $p_0$ and $p_c$ .

The current implementation also stores initialization bookkeeping variables in FEM runs: InitOverride\_R, InitOverride\_RS, InitOverride\_OCR, and OCR\_effective.

### 1.4 Three-Surface Framework

Zhao et al. (2005) define radial mapping between the three homologous surfaces in the  $p$ - $q$  plane:

$$R = \frac{p}{\tilde{p}} = \frac{q}{\tilde{q}}, \quad R_n = \frac{p^*}{\tilde{p}} = \frac{q^*}{\tilde{q}} \quad (2)$$

where:

- $(p, q)$  is the stress point on the **subloading** surface,
- $(p^*, q^*)$  is the image point on the **normal** surface, and
- $(\tilde{p}, \tilde{q})$  is the image point on the **superloading** surface.

With homologous Cam-Clay-type surfaces,

$$\tilde{p}_c = \frac{p_c}{R_n}, \quad p_{\text{sub}} = R \tilde{p}_c = R \frac{p_c}{R_n} \quad (3)$$

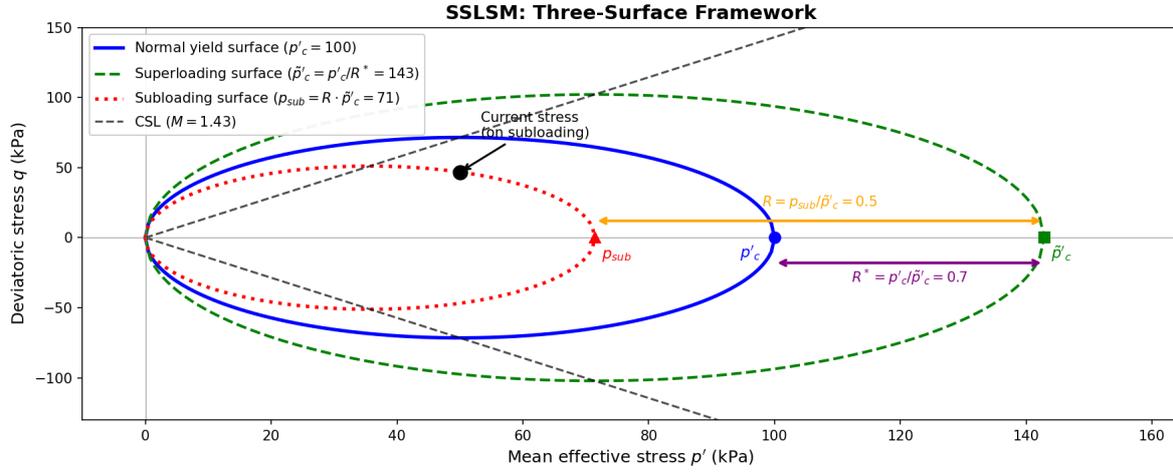


Figure 1: SSLSM three-surface schematic

and the UMAT uses the subloading yield function

$$f_{\text{sub}}(p, q) = q^2 + M(\theta)^2 p (p - p_{\text{sub}}) \quad (4)$$

Figure 1. Three homologous surfaces in SSLSM (schematic).

## 1.5 Elasticity and Hardening

The elastic bulk modulus follows Zhao et al. (2005):

$$K = \frac{v p}{\kappa} \quad (5)$$

with shear modulus

$$G = \frac{3(1 - 2\nu)}{2(1 + \nu)} K \quad (6)$$

The hardening variables are  $j = \{p_c, R, R_n\}$  and evolve as

$$dj = d\lambda B \quad (7)$$

with

$$B_1 = \frac{v p_c}{\lambda - \kappa} \frac{\partial f_{\text{sub}}}{\partial p}, \quad B_2 = -\frac{v M}{\lambda - \kappa} m \ln(R) d\varepsilon_d, \quad B_3 = \frac{v M a_{\text{sub}}}{\lambda - \kappa} R (1 - R_n) d\varepsilon_d \quad (8)$$

and

$$d\varepsilon_d = \sqrt{(1 - A_d) \left( \frac{\partial f_{\text{sub}}}{\partial p} \right)^2 + A_d \left( \frac{\partial f_{\text{sub}}}{\partial q} \right)^2} \quad (9)$$


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## 1.6 Initialization

Initialization is performed through the UMAT hook `initializeCustomVariable`, which conditions  $(p_c, R, R_n)$  from the current stress state and the supplied controls.

For an isotropic initial stress state:

- if `InitOnSubloading=1`, the initializer enforces

$$p_0 = R \frac{p_c}{R_n} \quad (10)$$

- if `InitOnSubloading=0`, the initializer uses the supplied OCR, R, and RS directly, with

$$p_c = \text{OCR } p_0 \quad (11)$$

If `UpdateVoidRatio=1`, the void ratio is rebuilt from a Cam-Clay-type compression relation:

$$v(p_0, p_c) = v_N - \lambda \ln p_c + \kappa \ln \left( \frac{p_c}{p_0} \right) \quad (12)$$


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## 1.7 Implementation Notes

- RS in the code stores the paper's  $R_n$ .
  - The UMAT expects the host to provide void ratio through the standard state-variable channel.
  - In FEM decks, any custom variable written during initialization must also appear in `Custom Variable=`.
  - The current FE tangent path uses an elastic Jacobian for robustness while the nonlinear stress update remains in `calculateStressIncrement()`.
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## 1.8 Single-Point Validation

The current documentation keeps one representative drained triaxial validation, matching the lean style used in `mits1model.md`.

- Input deck: [ssls<sub>m</sub>\\_triaxial\\_ocr\\_sensitivity.txt](#)

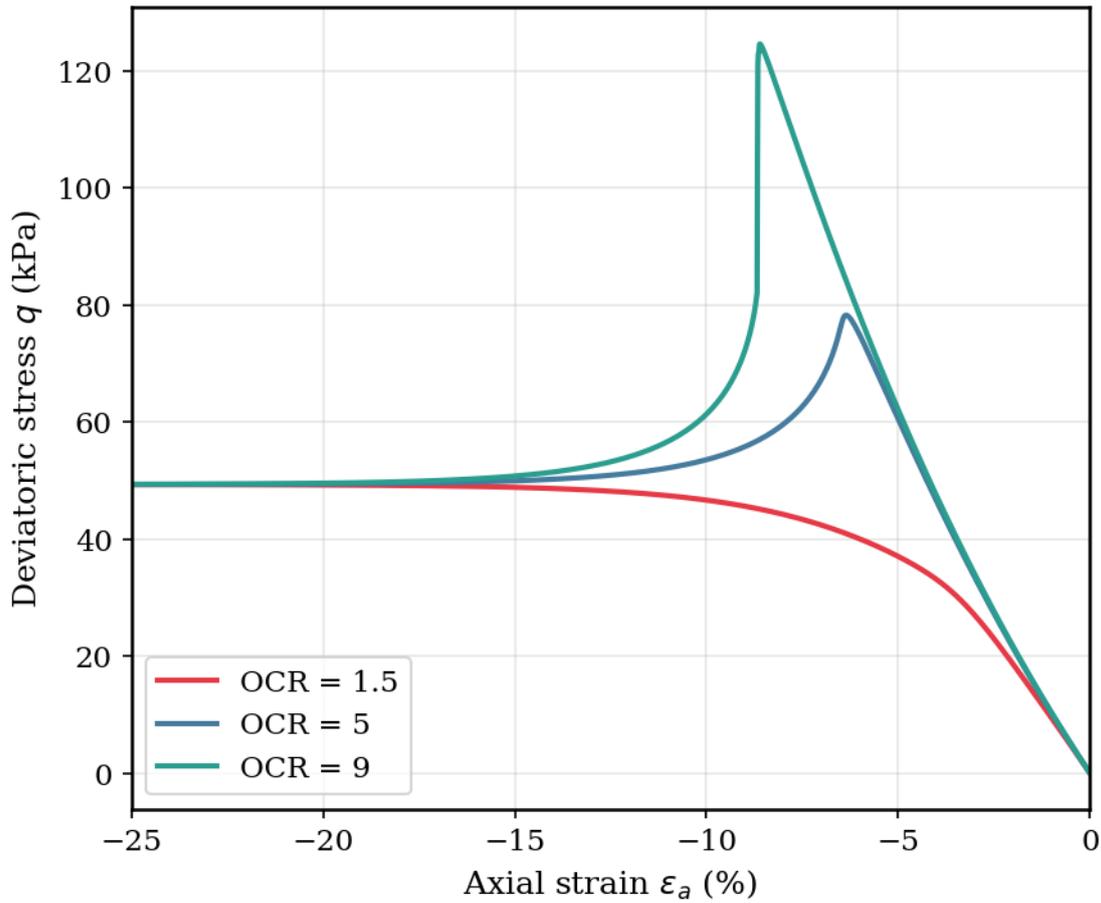


Figure 2: SSLSM OCR sensitivity in drained triaxial compression

- Loading mode: drained triaxial compression at constant radial stress
- Comparison shown: OCR sensitivity in  $q$ - $\epsilon_a$

Figure 2. Representative single-point SSLSM validation: drained triaxial compression with varying OCR.

## 1.9 FEM Footing Setup

A displacement-controlled strip-footing benchmark deck is provided for SSLSM:

- Input deck: `footing_strip_sslsm.txt`
- Runner: `run_sslsm_footing.py`

The benchmark uses:

- plane-strain half-domain geometry,

- prescribed vertical settlement over a footing-width node set at the ground surface, and
- reaction-force summation over the footing nodes to produce a load-settlement curve.

This setup is intended to mirror the displacement-controlled strip-footing workflow used elsewhere in the manual while avoiding the extra contact-model complexity of the rigid-footing examples.

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## 1.10 References

1. Zhao, J., Sheng, D., Sloan, S. W., & Abbo, A. J. (2005). *Explicit stress integration of complex soil models*. International Journal for Numerical and Analytical Methods in Geomechanics, 29, 1209-1229.

