AD FALCON API Manual

Void-Ratio Multiplier (k_e) for Unsaturated Permeability

Javad Ghorbani

March 14, 2026

Contents

1 Void-Ratio Multiplier (k_e) for Unsaturated Permeability

In unsaturated flow models, it is often useful to separate:

- a saturation-dependent relative permeability $k_r(S_e)$, typically bounded by $[0, 1]$, and
- a void-ratio-dependent permeability multiplier $k_e(e)$ that scales the hydraulic conductivity as the pore structure changes.

With anisotropic permeability, the effective permeability tensor components are applied in the form:

$$k_{\text{eff}} = k_{\text{aniso}}\ k_{sat}\ k_e(e)\ k_r(S_e)\ /\ \mu$$

where k_{aniso} is the directional multiplier from `@AnisotropicPerm:`, k_{sat} is the saturated permeability from `@Perm:`, and μ is the phase viscosity.

1.1 The $k_e(e)$ multiplier

When enabled, the void-ratio multiplier is computed from the current void ratio e and a reference void ratio e_{ref}:

$$k_e(e) = \frac{e^3(1 + e_{ref})^2}{e_{ref}^3(1 + e)^2}$$

This is applied only when $e_{ref} > 0$ and $e > 0$. Otherwise, $k_e(e) = 1$ (no void-ratio scaling).

1.2 Available models

Two permeability models provide $k_e(e)\, k_r(S_e)$ behavior:

- `VanGenuchtenKe`
- `BrooksCoreyKe`

Both models also support a lower bound `k_min` on the final multiplier to avoid numerically-zero conductivities:

$$\max\{k_e(e)\, k_r(S_e),\ k_{min}\}$$

1.3 Syntax

```
@Perm: VanGenuchtenKe m <m> k_sat <k_sat> [ke_ref <e_ref>] [k_min <k_min>]
@Perm: BrooksCoreyKe  lambda <lambda> k_sat <k_sat> [ke_ref <e_ref>] [k_min
<k_min>]
```

Notes:

- ke_ref and e_ref are accepted as synonyms for the reference void ratio.
- If ke_ref/e_ref is omitted (or set to 0), then $k_e(e) = 1$ and the model reduces to the corresponding saturation-only $k_r(S_e)$ form (with k_min still applied).

The multiplier $k_e(e)$ has the classical Kozeny-Carman void-ratio form. In FALCON it is combined with either the Mualem-van Genuchten or Brooks-Corey/Burdine relative-permeability functions, depending on the selected @Perm: model.

1.4 References

- Carman, P. C. (1937). *Fluid flow through granular beds*. Transactions of the Institution of Chemical Engineers, 15, 150-166.
- Mualem, Y. (1976). *A new model for predicting the hydraulic conductivity of unsaturated porous media*. Water Resources Research, 12(3), 513-522.
- van Genuchten, M. Th. (1980). *A closed-form equation for predicting the hydraulic conductivity of unsaturated soils*. Soil Science Society of America Journal, 44(5), 892-898. https://doi.org/10.2136/sssaj1980.03615995004400050002x
- Brooks, R. H., & Corey, A. T. (1964). *Hydraulic properties of porous media and their relation to drainage design*. Transactions of the ASAE, 7(1), 26-28.
- Burdine, N. T. (1953). *Relative permeability calculations from pore-size distribution data*. Journal of Petroleum Technology, 5(3), 71-78.