AD FALCON API Manual

# Anisotropic Permeability Model

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# 1 Anisotropic Permeability Model

The Anisotropic Permeability model allows specification of a full permeability tensor to account for directional variations in soil permeability. This is essential for modeling soils with layered structures, preferential flow paths, or other anisotropic characteristics.

## 1.1 Syntax

```
@AnisotropicPerm: <XX> <YY> <ZZ> <XY> <ZX> <ZY>
```

**Format**: Six permeability coefficients representing the full permeability tensor: - `XX`, `YY`, `ZZ`: Diagonal components (principal directions) - `XY`, `ZX`, `ZY`: Off-diagonal components (shear terms)

## 1.2 Mathematical Formulation



The anisotropic permeability tensor, $\mathbf{k}$, is a symmetric 3×3 tensor:

$$\mathbf{k} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}$$

where $k_{xy} = k_{yx}$, $k_{xz} = k_{zx}$, and $k_{yz} = k_{zy}$ due to symmetry.

### 1.2.1 Interaction with Relative Permeability Models

The anisotropic permeability model works in conjunction with isotropic permeability models specified via `@Perm:`. The final permeability tensor components are computed as:

$$k_{ij}^{final} = k_{ij}^{aniso} \times k_r(S_e, e) \times k_{sat}/\mu$$

where: - $k_{ij}^{aniso}$: Anisotropic scaling factor from `@AnisotropicPerm:` - $k_r(S_e, e)$: Relative permeability from the `@Perm:` model (depends on effective saturation $S_e$ and void ratio $e$). The void ratio effect is incorporated into $k_r$ through the void ratio correction factor $k_c$ when applicable (see Void Ratio Affected Constant Permeability, van Genuchten Permeability, and Brooks-Corey Permeability) - $k_{sat}$: Saturated permeability from the `@Perm:` model (constant, not affected by void ratio) - $\mu$: Fluid viscosity (water viscosity for liquid, gas viscosity for air)

### 1.2.2 For Coupled Analyses (Water Only)

In coupled analyses, only water permeability is computed:

$$k_{w,ij} = k_{ij}^{aniso} \times k_{rw}(S_e, e) \times k_{sat}/\mu_w$$

where $k_{rw}$ is the relative permeability for water from the `@Perm:` model, and $S_e = 1.0$ (fully saturated).

### 1.2.3 For Fully Coupled Analyses (Water and Air)

In fully coupled analyses, both water and air permeabilities are computed:

**Water permeability:**

$$k_{w,ij} = k_{ij}^{aniso} \times k_{rw}(S_e, e) \times k_{sat}/\mu_w$$

**Air permeability:**

$$k_{a,ij} = k_{ij}^{aniso} \times k_{rg}(S_e, e) \times k_{sat}/\mu_g$$

where: - $k_{rw}$: Relative permeability for water from the `@Perm:` model - $k_{rg}$: Relative permeability for gas from the `@Perm:` model - $S_e$: Effective saturation (computed from SWRC model) - $\mu_w$: Water viscosity - $\mu_g$: Gas viscosity

## 1.3 Parameter Specification

### 1.3.1 Isotropic Case

For isotropic permeability, use equal diagonal values and zero off-diagonal terms:

```
@AnisotropicPerm: 1.0 1.0 1.0 0.0 0.0 0.0
```

This is equivalent to not specifying `@AnisotropicPerm:` at all (the code defaults to isotropic behavior).

### 1.3.2 Transversely Isotropic Case

For transversely isotropic materials (e.g., horizontally layered soils), use:

```
@AnisotropicPerm: 1.0 1.0 0.1 0.0 0.0 0.0
```

This represents: - Horizontal permeability: $k_{xx} = k_{yy} = 1.0$ (reference) - Vertical permeability: $k_{zz} = 0.1$ (10% of horizontal) - No cross-coupling: $k_{xy} = k_{zx} = k_{zy} = 0.0$

### 1.3.3 Fully Anisotropic Case

For fully anisotropic materials, specify all six components:

```
@AnisotropicPerm: 1.0 0.5 0.2 0.1 0.05 0.02
```

This represents a general anisotropic permeability tensor with: - Principal components: $k_{xx} = 1.0$, $k_{yy} = 0.5$, $k_{zz} = 0.2$ - Off-diagonal components: $k_{xy} = 0.1$, $k_{zx} = 0.05$, $k_{zy} = 0.02$

## 1.4 Optional Specification

**Important**: The `@AnisotropicPerm:` directive is **optional**. When not specified: - The code defaults to isotropic permeability behavior - All permeability tensor components use the same value from the `@Perm:` model - The permeability tensor becomes: $k_{ij} = k_r(S_e, e) \times k_{sat}/\mu$ for $i = j$, and $k_{ij} = 0$ for $i \neq j$

## 1.5 Material Requirements

- **Coupled analyses (`*Coupled`)**: Requires `@Perm:` directive. Anisotropic permeability is optional.
- **Fully coupled analyses (`*FullyCoupled`)**: Requires `@Perm:` and `@SWRC:` directives. Anisotropic permeability is optional.

## 1.6 Examples

### 1.6.1 Example 1: Transversely Isotropic (Layered Soil)

```
% Materials
LayeredClay
@UMAT:/path/to/GCCUMAT.cpp /path/to/GCCUMAT.hpp Mechanical lambda=0.2
kappa=0.05 M=1.25
@Perm: VanGenuchten m 0.98 k_sat 1e-10
@AnisotropicPerm: 1.0 1.0 0.1 0.0 0.0 0.0
@PhaseChar: Solid rhos 2.7
@PhaseChar: Liquid rhow 0.997 K_l 2.25e6 l_vicosity 1e-6
%%%
```

Horizontal permeability is 10 times larger than vertical permeability.

### 1.6.2 Example 2: Fully Coupled with Anisotropic Permeability

```
% Materials
AnisotropicSoil
@UMAT:/path/to/GCCUMAT.cpp /path/to/GCCUMAT.hpp Mechanical lambda=0.2
kappa=0.05 M=1.25
@SWRC: Hysteretic alpha_1 0.5 n 1.4 m 0.5 omega_prime 0.1 alpha_2 0.2 bd
0.05 bw 0.05 SW_max 0.45 SW_min 0.05 b_s_c 0.01
@EffectiveStress: GhorbaniKodikara Beta1 0.55 Beta2 0.25
@Perm: BrooksCorey lambda 1.5 k_sat 1e-12
@AnisotropicPerm: 1.0 0.8 0.3 0.0 0.0 0.0
@PhaseChar: Solid rhos 2.7
@PhaseChar: Liquid rhow 0.997 K_l 2.25e6 l_vicosity 1e-6
```

```
@PhaseChar: Gas rhog 1.1e-3 k_g 1.01e2 g_vicosity 1.8e-5
%%%
```

Both water and air permeabilities will be anisotropic, with the same directional scaling factors applied to both phases.

### 1.7 Physical Interpretation

- **Diagonal components**: $k_{xx}$, $k_{yy}$, $k_{zz}$ represent permeability in the X, Y, and Z directions
- **Off-diagonal components**: $k_{xy}$, $k_{zx}$, $k_{zy}$ represent coupling between directions

For most geotechnical applications, off-diagonal terms are zero when the coordinate system aligns with the principal directions of anisotropy.

### 1.8 Dimensional Consistency

All anisotropic permeability coefficients are **dimensionless scaling factors**. The actual permeability values have dimensions **L²** (length squared), but the anisotropic coefficients multiply the base permeability from the `@Perm:` model, which already has the correct dimensions.

The final permeability tensor components have dimensions **L²/(M·T)** when divided by viscosity, or **L/T** when expressed as hydraulic conductivity, where: - **L**: length - **M**: mass - **T**: time

### 1.9 Related Documentation

- Permeability Overview - General permeability models
- Constant Permeability - Isotropic constant permeability
- Void Ratio Affected Constant Permeability - Void ratio effects
- van Genuchten Permeability - Van Genuchten relative permeability
- Brooks-Corey Permeability - Brooks-Corey relative permeability
- Materials - General material property definitions