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AD FALCON API Manual

NorSand Model (saturated)

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Contents

1 NorSand Model (saturated)

NorSand is a **critical-state sand model** with an image pressure `Pim` controlling the yield surface size. This page documents the **input syntax** and the **full mathematical formulation** used by the NorSand UMAT in this codebase.

1.1 Syntax

This model is configured in `% Materials` as a user-defined mechanical material. Use `@UMAT:` with category `Mechanical` and pass the parameters as `name=value` pairs.

Example (log CSL):

```
@UMAT: path/to/NorSandModelUMAT.cpp path/to/NorSandModelUMAT.hpp Mechanical
\
    CSLForm=0 Gref=20000 Pref=100 nG=0.6 Nu=0.10 \
    Gamma=1.115 LambdaCSL=0.076 Patm=100 \
    Mtc=1.40 N=0.30 Xtc=2.50 H0=30 Hpsi=200 \
    R=1.0 S=1 P_min=1e-6 FTOL=1e-6 \
    CustomVariable=Pim,Undrained
```

Example (power CSL):

```
@UMAT: path/to/NorSandModelUMAT.cpp path/to/NorSandModelUMAT.hpp Mechanical
\
    CSLForm=1 Gref=20000 Pref=100 nG=0.5 Nu=0.15 \
    Ca=0.90 Cb=0.14 Cc=0.15 Patm=100 \
    Mtc=1.28 N=0.30 Xtc=4.60 H0=100 Hpsi=625 \
    R=1.2 S=1 P_min=1e-6 FTOL=1e-6 \
    CustomVariable=Pim,Undrained
```

For readability, examples are wrapped across multiple lines; in input files you should write the full `@UMAT:` directive on a single line.

1.2 Material parameters

Parameters are passed as `name=value` pairs in the `@UMAT:` line.

Table 1. Material parameters

Symbol	Keyword in input	Units	Required	Description
–	`CSLForm`	–	✓	CSL form selector (`0` log, `1` power).
G_{ref}	`Gref`	stress	✓	Shear modulus reference value.
P_{ref}	`Pref`	stress	✓	Reference pressure for $G(p)$.
n_G	`nG`	–	✓	Shear modulus exponent.
ν	`Nu`	–	✓	Poisson's ratio.
Γ	`Gamma`	–	✓*	Log CSL intercept (only if `CSLForm=0`).
λ	`LambdaCSL`	–	✓*	Log CSL slope (only if `CSLForm=0`).
C_a	`Ca`	–	✓*	Power CSL intercept (only if `CSLForm=1`).
C_b	`Cb`	–	✓*	Power CSL coefficient (only if `CSLForm=1`).
C_c	`Cc`	–	✓*	Power CSL exponent (only if `CSLForm=1`).
p_{atm}	`Patm`	stress	✓	Normalizing pressure for power CSL.
M_{tc}	`Mtc`	–	✓	Triaxial-compression CSL ratio.
N	`N`	–	✓	Operating ratio parameter.
X_{tc}	`Xtc`	–	✓	Dilation limit parameter.
H_0	`H0`	–	✓	Hardening parameter.
H_ψ	`Hpsi`	–	✓	Hardening parameter.

Symbol	Keyword in input	Units	Required	Description
R	`R`	–	✓	OCR helper for initialization (must satisfy `R>=1`).
–	`S`	–	✓	Softening flag (`0` or `1`).
$P_{\min}$	`P_min`	stress	✓	Numerical lower bound for pressure-dependent terms.
–	`FTOL`	–	✓	Yield-surface tolerance.

* "Required" depends on `CSLForm`.

1.2.1 Optional integration controls

These keys are optional. If omitted, the UMAT defaults are used.

Keyword in input	Default	Description
`IntegrationScheme`	`0`	`0` explicit adaptive, `1` implicit substepping, `2` single-step return mapping.
`STOL`	`1e-4`	Local integration error tolerance for explicit adaptive substepping.
`DTMIN`	`1e-3`	Minimum adaptive substep size (in `(0,1]`).
`MaxSubsteps`	`200`	Maximum substeps for substepping schemes.
`MinImplicitSubsteps`	`2`	Minimum enforced implicit substeps.
`MaxNewtonIter`	`200`	Maximum Newton iterations.
`ExternalCompressionSign`	`-1`	`-1` = compression negative in the host (FALCON default), `+1` = compression positive.

1.3 Custom state variables

Declare custom variables using `CustomVariable=` in the `@UMAT:` line.

Name	Required	Meaning
`Pim`	recommended	Image pressure controlling the yield surface size.
`Undrained`	recommended	Drainage flag for the softening term (`1` = undrained, `0` = drained).

Optional diagnostics (declare via `CustomVariable=` only if you want them stored/visible for output):

- `iplas` (plastic step flag)
- `SubstepFailureFlag`

1.3.1 Recommended way to set `Undrained` (via `% Initial Assignments`)

If you declare `CustomVariable=Undrained`, you can set it with initial assignments:

```
% Initial Assignments
@Undrained: H 0.0 values 1   H 10.0 values 1
%%%
```

See Initial Assignments for the general syntax.

1.4 Stress invariants

Let $\boldsymbol{\sigma}$ be the effective stress tensor using FALCON's sign convention (tension positive). Define:

$$\sigma_m = \frac{1}{3}\,\mathrm{tr}(\boldsymbol{\sigma}), \qquad \boldsymbol{s} = \boldsymbol{\sigma} - \sigma_m \mathbf{I}$$

$$J_2 = \frac{1}{2}\,\boldsymbol{s} : \boldsymbol{s}, \qquad J_3 = \det(\boldsymbol{s})$$

The deviatoric stress magnitude (UMAT output column `q`) is

$$q = \sqrt{3J_2} \geq 0$$

and the mean pressure used in all NorSand equations is

$$p = -\sigma_m$$

The Lode angle $\theta \in [-\pi/6, \pi/6]$ is computed from J_2, J_3 (standard definition; see MohrModel for the invariant conventions used across the manual).

1.5 Elastic law

NorSand uses pressure-dependent isotropic elasticity:

$$G(p) = G_{\text{ref}} \left(\frac{\max(P_{\min}, p)}{P_{\text{ref}}} \right)^{n_G}$$

$$K(p) = \frac{2(1+\nu)}{3(1-2\nu)} G(p)$$

The elastic stiffness matrix in Voigt form is the standard isotropic tensor:

$$\mathbf{D}^e = \begin{bmatrix} \lambda_e + 2\mu & \lambda_e & \lambda_e & 0 & 0 & 0 \\ \lambda_e & \lambda_e + 2\mu & \lambda_e & 0 & 0 & 0 \\ \lambda_e & \lambda_e & \lambda_e + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \quad \lambda_e = K - \tfrac{2}{3}G, \ \mu = G$$

1.6 Critical state line (CSL) and state parameters

NorSand supports two CSL forms. Let e be the void ratio and define the CSL void ratio $e_c(p)$.

1.6.1 Log CSL (`CSLForm=0`)

$$e_c(p) = \Gamma - \lambda \ln(\max(P_{\min}, p))$$

with λ from `LambdaCSL`.

1.6.2 Power CSL (`CSLForm=1`)

$$e_c(p) = C_a - C_b \left(\frac{\max(P_{\min}, p)}{p_{\text{atm}}} \right)^{C_c}$$

Define the (current) state parameter

$$\psi = e - e_c(p)$$

and the **image** state parameter (evaluated at `Pim`)

$$\psi_i = e - e_c(p_{im})$$

1.7 Lode-angle dependent critical friction ratio

The Lode-angle dependent critical friction ratio is

$$M(\theta) = M_{tc}\left(1 - \frac{M_{tc}}{3 + M_{tc}}\cos\left(\tfrac{3}{2}\theta + \tfrac{\pi}{4}\right)\right)$$

We denote $M_c = M(\theta)$.

1.8 Mapped dilation limit parameter

NorSand maps X_{tc} to a pressure-dependent limit parameter Ξ:

$$\Xi = \frac{X_{tc}}{1 - \lambda(p_{im})\, X_{tc}/M_{tc}}$$

where:

- for log CSL: $\lambda(p_{im}) = \lambda$ (constant),
- for power CSL:

$$\lambda(p_{im}) = -\left.\frac{de_c}{d\ln p}\right|_{p=p_{im}} = C_b\, C_c\left(\frac{\max(P_{\min}, p_{im})}{p_{\text{atm}}}\right)^{C_c}$$

1.9 Operating friction ratio and dilatancy

Let

$$\eta = \frac{q}{p}$$

The operating friction ratio M_i depends on M_c, ψ_i, and Ξ:
Dense side ($\psi_i < 0$):

$$M_i = M_c\left(1 + \frac{N\,\Xi}{M_{tc}}\psi_i\right)$$

Loose side ($\psi_i \geq 0$):

- default (extended Dafalias form):

$$M_i = M_c\left(1 - \frac{N\,\Xi}{M_{tc}}\psi_i\right)$$

- optional loose-side switch (Taylor–Bishop): if you input `Mtc < 0` in the UMAT parameters, the implementation uses $M_i = M_c$ for $\psi_i \geq 0$.

The plastic dilatancy function is

$$D_p = M_i - \eta$$

1.10 Yield function

The NorSand yield function is (compression-positive $p > 0$ form):

$$f(\boldsymbol{\sigma}, p_{im}, e) = q - \eta_y \, p$$

with the yield stress ratio

$$\eta_y = M_i \left(1 + \ln \left(\frac{p_{im}}{p} \right) \right)$$

The stress state is:

- **elastic** if $f \le \mathrm{FTOL}$
- **plastic** if $f > \mathrm{FTOL}$

1.11 Plastic potential and flow rule (stress–dilatancy form)

The UMAT uses a stress–dilatancy flow rule consistent with:

$$\frac{d\varepsilon_v^p}{d\varepsilon_q^p} = D_p$$

where:

- $\varepsilon_v^p = \varepsilon_{xx}^p + \varepsilon_{yy}^p + \varepsilon_{zz}^p$
- the implementation uses $\varepsilon_p^p = \varepsilon_v^p / 3$ as the p-conjugate strain invariant,
- and $d\varepsilon_q^p = d\lambda$ (because $\partial f / \partial q = 1$).

Accordingly, the plastic potential gradient is constructed so that:

$$\frac{\partial g}{\partial p} = \frac{D_p}{3}, \qquad \frac{\partial g}{\partial q} = 1$$

1.12 Hardening law for `Pim`

NorSand evolves the image pressure with plastic shearing:

$$\dot{p}_{im} = B_{\mathrm{iso}} \, \dot{\varepsilon}_q^p$$

The state-dependent hardening modulus is

$$H = H_0 - H_\psi \, \psi$$

and the hardening limit stress is

$$p_{mx} = p \, \exp\left(-\frac{X_{tc}}{M_{i,tc}} \psi \right)$$

where $M_{i,tc}$ is the operating friction ratio evaluated at triaxial compression ($\theta = \pi/6$).

The base hardening term is:

$$B_{\text{iso,base}} = H \left(\frac{p}{p_{im}} \right) \left(\frac{M_i}{M_{i,tc}} \right) (p_{mx} - p_{im})$$

1.12.1 Optional softening term (S)

If `S=1`, a softening term is applied under **undrained** conditions when $D_p > 0$:

$$S_{\text{soft}} = \omega \left(\frac{\eta}{M_i} \right) \left(\frac{K}{p} \right) D_p \, p_{im}$$

with

$$\omega = 1 - \lambda(p) \, X_{tc}/M_{tc}$$

and $\lambda(p)$ evaluated at the **current** pressure p (constant for log CSL; local slope for power CSL).

The final hardening term used by the UMAT is:

$$B_{\text{iso}} = B_{\text{iso,base}} - S_{\text{soft}}$$

1.12.2 Consistency hardening modulus

The scalar hardening modulus appearing in the consistency condition is:

$$K_p = -\frac{\partial f}{\partial p_{im}} \frac{d p_{im}}{d\lambda} = \left(M_i \frac{p}{p_{im}} \right) B_{\text{iso}}$$

1.13 Integration options

The UMAT provides three integration schemes selectable by the optional parameter `IntegrationScheme`:

- `0`: explicit adaptive substepping
- `1`: implicit substepping
- `2`: single-step return mapping

The host may also request a runtime override via the per-step flag `initializationMethodFlag` in `UMATBase::InputData`:

- `1`: single-step return mapping
- `2`: implicit substepping
- `3`: explicit adaptive substepping

1.14 Initialization and post-equilibrium conditioning (`Pim`)

After initialization (or after a geostatic/body-force equilibrium update), the stress state and void ratio may not be consistent with the currently stored `Pim`. The NorSand UMAT conditions `Pim` so the current stress state lies **on or inside** the yield surface:

- If `Pim` is missing, it is initialized from the current stress state and then enlarged using `R` (OCR helper).
- If `Pim` is present, it is only increased if needed (never shrunk).

This prevents starting a step from an overstressed state and avoids abrupt numerical corrections at the first constitutive update.

1.15 Example input files (verification-style)

The following example inputs are provided under `docs/Falcon_inputs/norsand/`:

Case	Implicit	Explicit
TXU dense (log CSL)	TXU_dense_log_implicit.txt	TXU_dense_log_explicit.txt
TXU loose (log CSL)	TXU_loose_log_implicit.txt	TXU_loose_log_explicit.txt
TXD dense (log CSL)	TXD_dense_log_implicit.txt	TXD_dense_log_explicit.txt
TXD loose (log CSL)	TXD_loose_log_implicit.txt	TXD_loose_log_explicit.txt
UDSS dense (log CSL, S=0)	UDSS_dense_log_S0_implicit.txt	UDSS_dense_log_S0_explicit.txt
UDSS dense (log CSL, S=1)	UDSS_dense_log_S1_implicit.txt	UDSS_dense_log_S1_explicit.txt
UDSS loose (log CSL, S=0)	UDSS_loose_log_S0_implicit.txt	UDSS_loose_log_S0_explicit.txt
UDSS loose (log CSL, S=1)	UDSS_loose_log_S1_implicit.txt	UDSS_loose_log_S1_explicit.txt
TXU dense (power CSL)	TXU_dense_pow_implicit.txt	TXU_dense_pow_explicit.txt

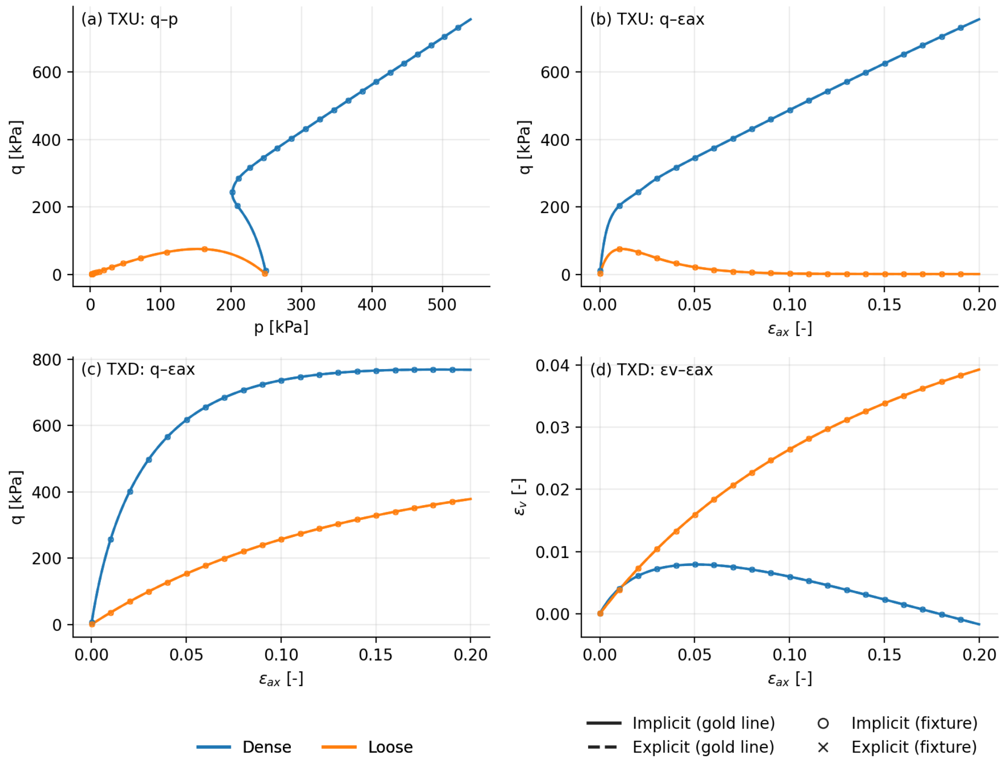

Figure 1: NorSand verification overlay: TXU/TXD

Case	Implicit	Explicit
TXU loose (power CSL)	TXU_loose_pow_implicit.txt	TXU_loose_pow_explicit.txt

---

1.16 Simulation results

Overlay of reference verification curves (lines) and fixture-based simulation outputs (markers) for undrained/drained triaxial tests. Solid = implicit (verification), dashed = explicit (verification). Markers: o = implicit (fixture), x = explicit (fixture). Colors: dense vs loose.

Overlay of reference verification curves (lines) and fixture-based simulation outputs (markers) for UDSS tests (S=0 and S=1). Solid = implicit (verification), dashed = explicit (verification). Markers: o = implicit (fixture), x = explicit (fixture). Colors: dense vs loose; lighter tone indicates S=1.

Overlay of reference verification curves (lines) and fixture-based simulation outputs (mark-

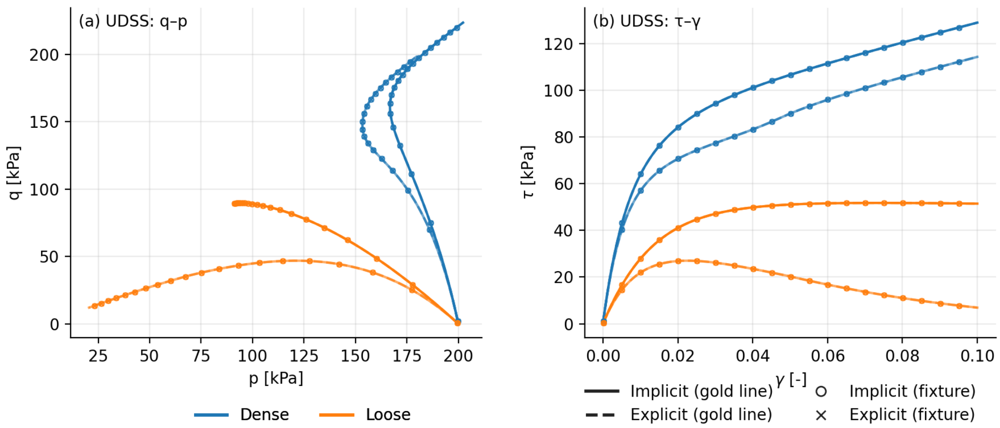

Figure 2: NorSand verification overlay: UDSS

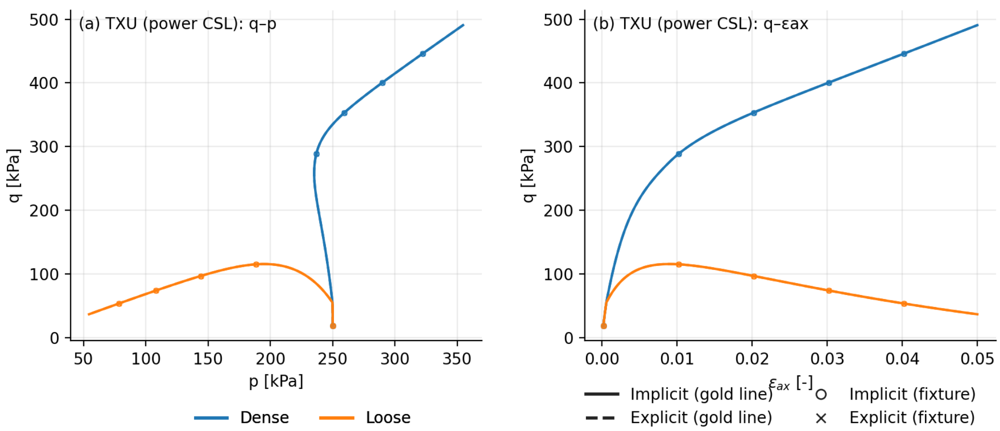

Figure 3: NorSand verification overlay: power CSL

ers) for the power-form CSL examples. Solid = implicit (verification), dashed = explicit (verification). Markers: o = implicit (fixture), x = explicit (fixture). Colors: dense vs loose.

1.16.1 Parameter sensitivity (TXU dense, log CSL)

One-at-a-time sensitivity about the TXU dense (log CSL) base case. Each panel varies one parameter by multipliers (0.8×, 1.0×, 1.2×) while holding all other inputs fixed; plotted in q–ϵa space.

Same sensitivity runs as above, plotted in q–p' space (p' is the effective mean stress output column p).

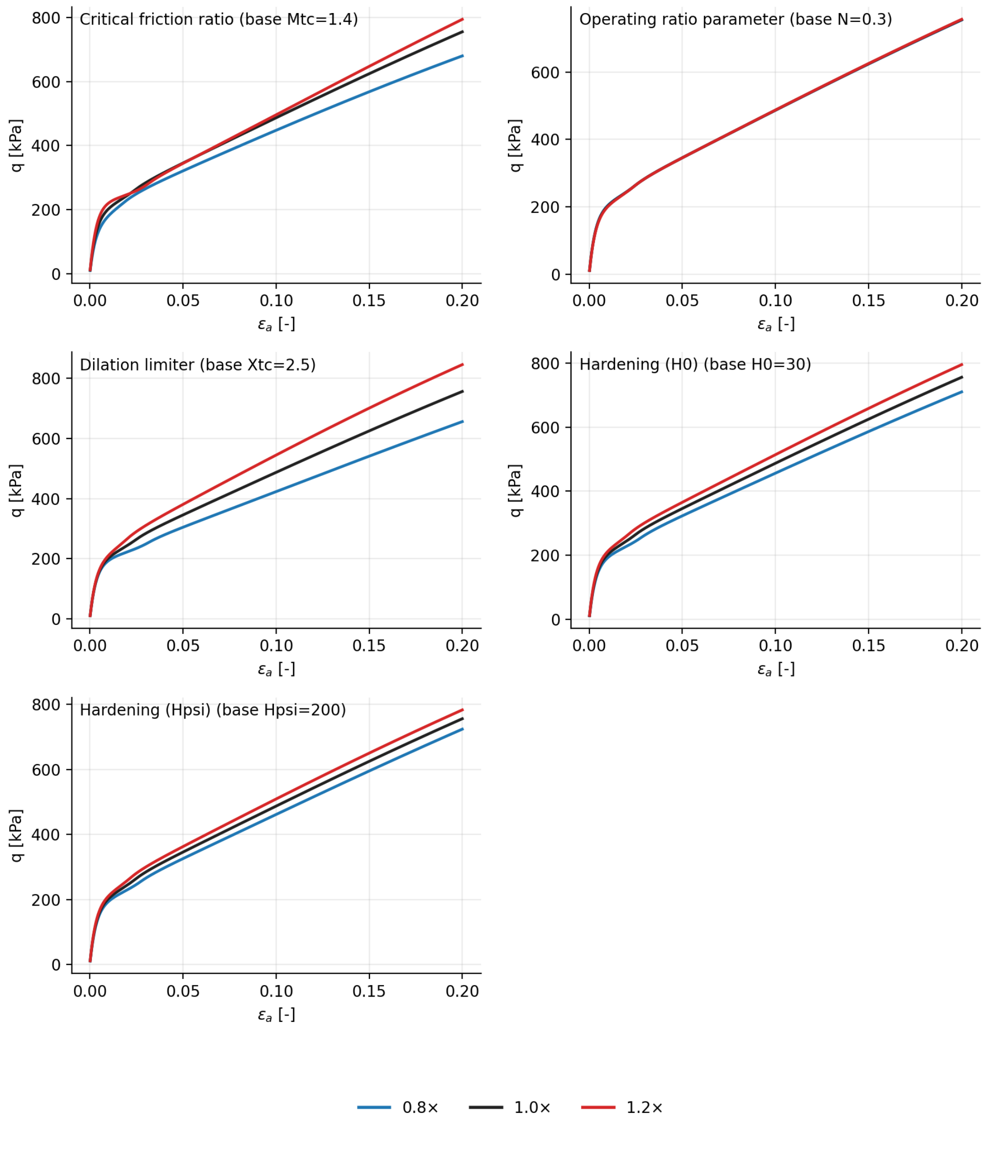

Figure 4: NorSand sensitivity: q–ϵa

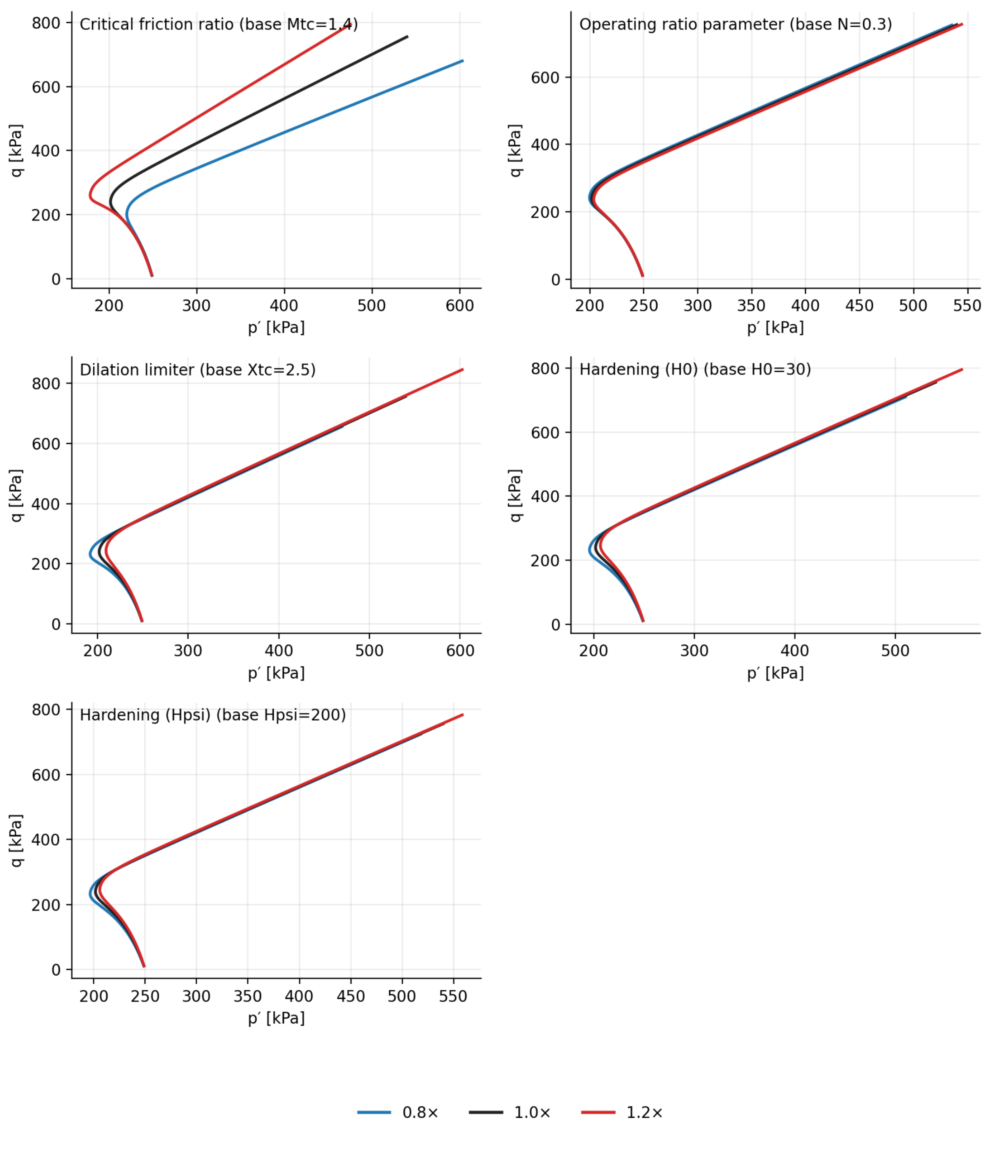

Figure 5: NorSand sensitivity: q–p′