AD FALCON API Manual

Theory of Mohr–Coulomb Plasticity

Javad Ghorbani

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1 Theory of Mohr–Coulomb Plasticity

This note restates Mohr–Coulomb (M–C) plasticity with the smooth Abbo–Sloan approximation.

1.1 Syntax

This model is configured in `% Materials` as a user-defined mechanical material. Use `@UMAT:` with category `Mechanical` and pass the parameters as `name=value` pairs.

Example:

```
@UMAT: path/to/mohr.cpp path/to/mohr.hpp Mechanical E=1e8 Nu=0.3 Phi=30
Cohesion=10e3 Psi=0
```

Optional parameters:

- `DilationPvCap` (dilatancy cap): caps the accumulated plastic volumetric strain at which dilatancy is switched off (omit it, or set it ≤ 0, to disable).

Example with a dilatancy cap:

```
@UMAT: path/to/mohr.cpp path/to/mohr.hpp Mechanical E=1e8 Nu=0.3 Phi=30
Cohesion=10e3 Psi=10 DilationPvCap=0.02
```

Use the parameter names shown in the table below.

1.2 Material parameters

Symbol	Keyword in input	Units	Required	Description
E	`E`	stress	✓	Young's modulus.
ν	`Nu`	–	✓	Poisson's ratio.
ϕ'	`Phi`	°	✓	Friction angle (effective stress).
c'	`Cohesion`	stress	✓	Cohesion (effective stress).
ψ	`Psi`	°	✓	Dilatancy angle for non-associated flow.
a	`RoundingParam`	–	✓	Hyperbolic rounding parameter.

Symbol	Keyword in input	Units	Required	Description
STOL	STOL	–	✓	Stress integration tolerance.
FTOL	FTOL	–	✓	Yield-surface tolerance.
$\varepsilon_v^{p,\,cap}$	DilationPvCap	strain	×	Optional cap on the accumulated plastic volumetric strain (positive in tension) at which the dilatancy angle is set to zero (disabled if ≤ 0).

1.3 Effective stress for unsaturated soils

Let the **net stress** and **matric suction** be

$$\boldsymbol{\sigma}_{\text{net}} = \boldsymbol{\sigma} + p_a\,\mathbf{I}, \qquad s = p_a - p_w$$

With Bishop-type weighting $\chi \in [0,1]$, the **effective stress** used throughout the model is

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma}_{\text{net}} - \chi\, s\,\mathbf{I} \tag{1}$$

so that suction contributes to strength through the **suction stress** $\sigma_{\text{suc}} = \chi\, s$. This recovers the familiar saturated and dry limits and is the basis for calibration from shear tests.

1.3.1 Enriched $\chi(S_w)$ (near-dry attenuation)

To capture the observed **peak and attenuation** of suction-driven strength toward the dry end, χ is enriched as a **multiplicative function of degree of saturation** S_w. The Ghorbani and Kodikara (2024) formulation provides a flexible representation:

$$\chi(S_w) = S_w^{\left(\frac{\beta_1}{S_w^{\beta_2}}\right)} \tag{2}$$

with two material parameters (β_1, β_2) controlling the shape. This preserves $\chi \to 0$ as $S_w \to 0$ and $\chi \to 1$ as $S_w \to 1$, while allowing a tunable near-dry decay of σ_{suc}. Set $\beta_1 = 1$ and $\beta_2 = 0$ to recover the classical $\chi = S_w$ (Bishop's model).

For detailed information, see Effective Stress Model – Ghorbani and Kodikara (2024).

Note on coupled saturated analyses: In fully saturated coupled analyses where

$S_w = 1$, the effective stress formulation reduces to Terzaghi's classical definition ($\chi = 1$), and the model operates as a standard Mohr–Coulomb plasticity in terms of effective stress. See Coupled Analysis for details on saturated formulations.

1.4 Stress invariants

Mean effective stress p (compression positive in geomechanics; here **remember stresses are tension-positive**, so $p = -\sigma_m$ if you prefer compression-positive):

$$\sigma_m = \tfrac{1}{3}\,\mathrm{tr}\,\boldsymbol{\sigma}', \quad \mathbf{s} = \boldsymbol{\sigma}' - \sigma_m \mathbf{I}, \quad J_2 = \tfrac{1}{2}\,\mathbf{s} : \mathbf{s}, \quad J_3 = \det(\mathbf{s}), \quad \theta = \tfrac{1}{3}\arcsin\Big(-\tfrac{4.5\,J_3}{\sqrt{3}\,J_2^{3/2}}\Big)$$

1.5 Smooth Mohr–Coulomb yield (Abbo–Sloan)

The classical Mohr–Coulomb criterion exhibits singular corners on the deviatoric plane. To enable robust numerical integration, we adopt the **hyperbolic smooth approximation** proposed by Abbo and Sloan (1995), which effectively rounds these angular corners while maintaining close fidelity to the original hexagonal surface. The yield function is expressed in stress invariants as:

$$f(\boldsymbol{\sigma}') = \sqrt{J_2}\,K(\theta) + \sigma_m \sin\phi' - c' \cos\phi' = 0 \tag{3}$$

where:
- ϕ' is the friction angle
- c' is the cohesion
- θ is the **Lode angle**, defined as

$$\theta = \tfrac{1}{3}\sin^{-1}\left(-\tfrac{3\sqrt{3}}{2}\,\tfrac{J_3}{J_2^{3/2}}\right)$$

The function $K(\theta)$ is defined **piecewise** to blend two hyperbolic branches across a **transition angle** θ_T:

$$K(\theta) = \begin{cases} A - B\sin(3\theta) & \text{if } |\theta| > \theta_T \\ \cos\theta - \tfrac{1}{\sqrt{3}}\sin\theta\,\sin\phi' & \text{if } |\theta| \le \theta_T \end{cases} \tag{4}$$

with parameters:

$$A = \tfrac{1}{3}\cos\theta_T\left(3 + \tan\theta_T\tan(3\theta_T) + (\tan(3\theta_T) - 3\tan\theta_T)\,\tfrac{1}{\sqrt{3}}\sin\phi'\,\sin\phi'\right) \tag{5}$$

$$B = \tfrac{1}{3\cos(3\theta_T)}\left(\sin\theta\sin\theta_T + \tfrac{1}{\sqrt{3}}\cos\theta\sin\phi'\right) \tag{6}$$

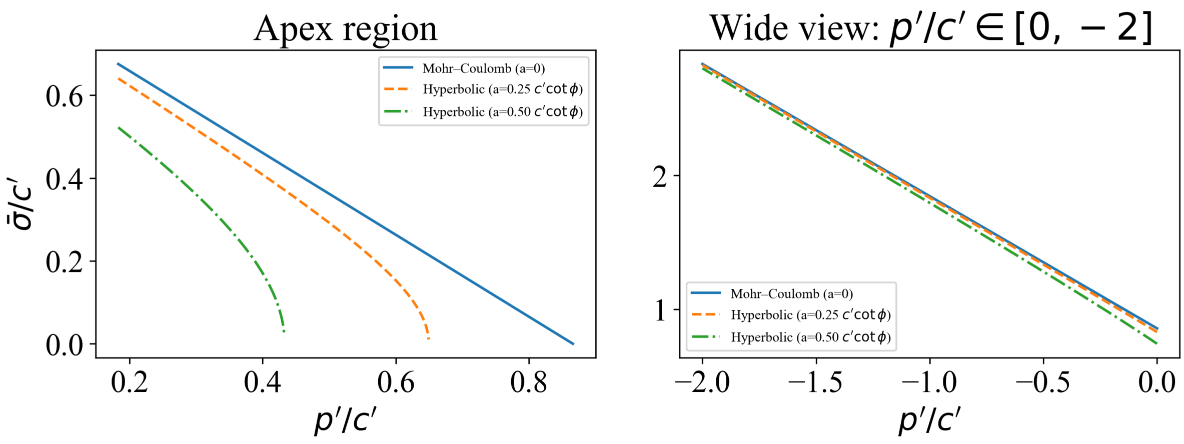

Figure 1: Effect of apex smoothing on the Mohr-Coulomb yield surface

1.5.1 Rounding parameter and transition angle

The **rounding parameter** a controls how closely the smooth surface matches the exact Mohr–Coulomb hexagon:
- For $a \leq 0.25$, the approximation is nearly indistinguishable from the original M–C surface, but $\cot\phi'$ must be small to avoid numerical ill-conditioning.
- As $a \to 0$, the hyperbolic surface converges to the hexagonal M–C criterion with sharp corners.
- A typical choice is $a \approx 0.05 \cot\phi'$, balancing accuracy with numerical stability.

The **transition angle** θ_T is selected slightly below the M–C corner angle to ensure smooth blending and avoid singularities near the corners. In FALCON, θ_T is set to a default value of **29.5°** and is not user-adjustable.

> **Note**: In principal stress space, this formulation recovers the familiar triaxial compression and extension meridians. The smooth approximation removes apex and edge singularities, enabling consistent tangent operators for implicit stress integration.

Figure 1.: The hyperbolic smoothing adopted in FALCON.

1.6 Plastic potential (non-associated)

Non-associated flow uses the same structure with the **dilatancy** ψ angle replacing the friction angle.

1.6.1 Optional dilatancy cap (`DilationPvCap`)

To prevent unbounded volumetric expansion in highly dilatant materials, an optional cap can be applied to the accumulated plastic volumetric strain (positive in tension):

- Define `DilationPvCap > 0` to activate. If omitted or set ≤ 0, the cap is disabled and the dilatancy remains equal to the prescribed ψ.
- Let the accumulated plastic strains be $\varepsilon^p_{xx}, \varepsilon^p_{yy}, \varepsilon^p_{zz}$. The model monitors the plastic volumetric strain $\varepsilon^p_v = \varepsilon^p_{xx} + \varepsilon^p_{yy} + \varepsilon^p_{zz}$ (positive in tension). When ε^p_v reaches `DilationPvCap`, the effective dilatancy used in the plastic potential is set to zero for the remainder of the analysis (i.e., $\psi_{\text{eff}} = 0$ afterward).

Practical guidance: - Smaller caps (e.g., `DilationPvCap = 0.02`) shut off dilatancy earlier, limiting void ratio growth and peak strength from dilation. - Larger caps (e.g., `DilationPvCap = 0.05`) allow more dilation before switching to zero-dilatancy flow. - With `DilationPvCap ≤ 0`, the model behaves as standard non-associated M–C with constant ψ.

Influence on void ratio and response The following results are obtained from **saturated triaxial drained compression** tests. The test conditions are:
- **Drainage**: Fully drained (constant pore pressure, fully saturated)
- **Loading**: Axial compression with constant radial effective stress (conventional triaxial compression)
- **Initial state**: Isotropic effective stress of -50 kPa (compression), initial void ratio $e_0 = 0.35$
- **Material parameters**: $E = 100$ MPa, $\nu = 0.30$, $c' = 1$ kPa, $\phi' = 30^\circ$, $\psi = 10^\circ$
- **Loading path**: Axial strain increment $\Delta\varepsilon_{yy} = -1.0 \times 10^{-4}$ per step (compression negative), maintaining $\Delta q/\Delta p' = 3$ (constant radial stress)

Figure 2. Void ratio evolution for multiple analyses: no dilatancy ($\psi = 0$), constant dilatancy ($\psi = 10^\circ$), and capped dilatancy ($\psi = 10^\circ$ with DilationPvCap = 0.02 and 0.05). Smaller caps arrest dilation earlier, resulting in lower final void ratios.

Figure 3. Mechanical response comparison under the same paths. Enabling dilatancy ($\psi > 0$) increases peak strength due to volumetric expansion; applying a cap reduces that effect beyond the cap, aligning post-peak trends closer to the non-dilatant response.

1.7 Elastic law

The model uses **linear isotropic elasticity** with constant Young's modulus E and Poisson's ratio ν. The elastic bulk and shear moduli are:

$$K = \frac{E}{3(1 - 2\nu)} \tag{7}$$

$$G = \frac{E}{2(1 + \nu)} \tag{8}$$

The elastic compliance tensor is standard:

$$
\mathbf{D}^e = \begin{bmatrix} K + \frac{4}{3}G & K - \frac{2}{3}G & K - \frac{2}{3}G & 0 & 0 & 0 \\ K - \frac{2}{3}G & K + \frac{4}{3}G & K - \frac{2}{3}G & 0 & 0 & 0 \\ K - \frac{2}{3}G & K - \frac{2}{3}G & K + \frac{4}{3}G & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{bmatrix} \tag{9}
$$

For plane strain (2D), the stress–strain relation reduces to a 3×3 sub-block corresponding to $[\sigma_{xx}, \sigma_{yy}, \sigma_{xy}]$ and $[\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}]$.

1.8 References (selection)

- Abbo, A. & Sloan, S. (1995). *A smooth hyperbolic approximation to the Mohr–Coulomb yield criterion.*