AD FALCON API Manual

Unified Clay and Sand Model (CASM)

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Contents

1 Unified Clay and Sand Model (CASM)

CASM is a **critical-state model** (Yu, 1998) that unifies clay-like and sand-like responses by combining:

- A flexible yield surface controlled by the exponent n
- Rowe-type stress–dilatancy for **non-associated** flow
- A single isotropic hardening variable controlling the cap size

This implementation is **saturated-only**; for unsaturated analyses see the GCC model.

1.1 Syntax

This model is configured in `% Materials` as a user-defined mechanical material. Use `@UMAT:` with category `Mechanical` and pass the parameters as `name=value` pairs.

Example:

```
@UMAT: path/to/CASMModelUMAT.cpp path/to/CASMModelUMAT.hpp Mechanical \
    Phi=30 Lambda=0.15 Kappa=0.03 Nu=0.25 Alpha=0.8 SSC=2 SPR=1.2 \
    P_min=0.1 DefaultIsoHardening=500 v_N=2.0 STOL=1e-5 FTOL=1e-6 LTOL=1e-6
\
    CustomVariable=IsotropicHardening
```

For readability, this example is wrapped across multiple lines; in input files you should write the full `@UMAT:` directive on a single line.

Use the parameter names shown in the tables below.

Notes:

- This UMAT expects exactly the required parameter set shown in the table; additional `name=value` pairs are not supported.
- Optional `OCR` is provided as a custom state variable (not as a UMAT parameter).

1.2 Material parameters

Table 1. Material parameters and their descriptions

Symbol	Keyword in input	Units	Required	Description
ϕ'	`Phi`	°	✓	Critical-state friction angle.

Symbol	Keyword in input	Units	Required	Description
λ	`Lambda`	–	✓	Virgin compression index (slope of NCL in v–$\ln p$ space). Must satisfy $\lambda > \kappa$.
κ	`Kappa`	–	✓	Swelling/reloading index. Must be positive and less than λ.
ν	`Nu`	–	✓	Poisson's ratio. Must satisfy $-1 < \nu < 0.5$.
α	`Alpha`	–	✓	Deviatoric shape parameter used in F_θ.
n	`SSC`	–	✓	Yield exponent controlling shape (CASM notation n).
r	`SPR`	–	✓	Spacing ratio (input must satisfy **SPR > 1**). Internally $r^* = 1/\ln(r)$.
v_N	`v_N`	–	✓	Specific volume at $p = 1$ (same convention as the code).
$P_{\min}$	`P_min`	stress	✓	Lower bound used inside elastic moduli to prevent $K, G \to 0$ at very small p. Must be positive.
a_0	`DefaultIso Hardening`	stress	✓	Additive floor used by post-equilibrium conditioning (see below). Must be positive.

Symbol	Keyword in input	Units	Required	Description
`STOL`	`STOL`	–	✓	Substepping/integration tolerance.
`FTOL`	`FTOL`	–	✓	Yield-surface tolerance used for branching and drift correction.
`LTOL`	`LTOL`	–	✓	Load–unload detection tolerance.

Notes

- **No `OCRControlled` and no `OverBurdenPressure`** parameters are used by the current CASM UMAT (unlike the unsaturated GCC model).
- **SPR validation:** because the code uses $r^* = 1/\ln(r)$, the input must satisfy $r > 1$ to avoid division by zero or negative values.

1.2.1 Constructor validation

The UMAT constructor performs the following checks and throws `std::invalid_argument` if any fails:

Condition	Rationale
$\kappa > 0$	Ensures positive elastic stiffness.
$\lambda > \kappa$	Required for physically meaningful plastic hardening.
$P_{\min} > 0$	Prevents zero/negative modulus floor.
$-1 < \nu < 0.5$	Valid Poisson's ratio bounds.
$a_0 > 0$ (`DefaultIsoHardening`)	Prevents unphysical zero cap floor.
$r > 1$ (SPR)	Ensures $\ln(r) > 0$ so r^* is finite and positive.

1.3 Custom state variables

Name	Required	Meaning
`Isotropic Hardening`	✓	The saturated cap size variable used by CASM (denoted σ_{mc0} in the code, equivalent to p_0 in p–q space).

Name	Required	Meaning
OCR	×	Optional multiplier used only during **post-equilibrium conditioning** (acts like b_0). Default is 0.

1.4 Elastic law

The UMAT uses pressure-dependent isotropic elasticity with stress-dependent tangent moduli:

$$K = v\,\frac{\max(P_{\min},\ p)}{\kappa}, \qquad v = 1 + e \tag{1}$$

$$G = \frac{3\,(1-2\nu)}{2\,(1+\nu)}\,K \tag{2}$$

where $p = -\sigma_m$ is the compressive mean pressure, e is the void ratio and $P_{\min} > 0$ is a numerical lower bound that prevents unbounded compliance at very small stresses.

The elastic stiffness matrix is

$$\mathbf{D}^e = \begin{bmatrix} \lambda_e + 2\mu & \lambda_e & \lambda_e & 0 & 0 & 0 \\ \lambda_e & \lambda_e + 2\mu & \lambda_e & 0 & 0 & 0 \\ \lambda_e & \lambda_e & \lambda_e + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \quad \lambda_e = K - \tfrac{2}{3}G,\ \mu = G \tag{3}$$

1.5 Yield surface

The CASM yield function (in compressive $p > 0$ form) is:

$$f = \left(\frac{q\,F_\theta}{M\,p}\right)^n + r^* \ln\left(\frac{p}{p_0}\right) = 0 \tag{4}$$

where:

- M is the critical state slope: $M = \frac{6\sin\phi'}{3-\sin\phi'}$
- p_0 is the isotropic hardening/cap variable (stored as `IsotropicHardening`)
- n is the yield exponent (SSC)
- $r^* = 1/\ln(r)$ with $r =$ SPR

The Lode-angle factor is

$$F_\theta = \frac{1}{\alpha}\left[\frac{1+\alpha^4-(1-\alpha^4)R_\theta}{2}\right]^{1/4}, \qquad R_\theta = -\frac{3\sqrt{3}\,J_3}{2J_2^{3/2}} \tag{5}$$

1.6 Plastic potential

CASM uses **non-associated flow** based on Rowe-type stress–dilatancy. The implementation follows the UMAT's internal equations for $\partial g/\partial\sigma$ and returns the plastic direction through `GradientOfPlasticToSigma()`.

1.7 Hardening law

The cap variable grows with plastic volumetric strain:

$$p_0 = p_{00}\,\exp\left(\frac{\varepsilon_v^p}{\lambda-\kappa}\right) \tag{6}$$

The plastic modulus K_p is computed from the chain rule:

$$K_p = -\frac{\partial f}{\partial p_0}\cdot B_{\text{iso}}, \qquad B_{\text{iso}} = \frac{\partial g}{\partial p}\frac{\partial p_0}{\partial \varepsilon_v^p} \tag{7}$$

1.8 Elastoplastic tangent

Loading on the yield surface enforces the consistency condition

$$\frac{\partial f}{\partial \boldsymbol{\sigma}} : d\boldsymbol{\sigma} + \frac{\partial f}{\partial p_0}\,dp_0 = 0 \tag{8}$$

yielding the plastic multiplier

$$d\lambda = \frac{\dfrac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbf{D}^e d\boldsymbol{\varepsilon}}{K_p + \dfrac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbf{D}^e : \dfrac{\partial g}{\partial \boldsymbol{\sigma}}} \tag{9}$$

The elastoplastic tangent is

$$\mathbf{D}_{ep} = \mathbf{D}^e - \frac{\mathbf{D}^e \frac{\partial g}{\partial \boldsymbol{\sigma}} \otimes \frac{\partial f}{\partial \boldsymbol{\sigma}} \mathbf{D}^e}{K_p + \frac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbf{D}^e : \frac{\partial g}{\partial \boldsymbol{\sigma}}} \tag{10}$$

1.9 Post-equilibrium conditioning (saturated)

After a geostatic/equilibrium step, the host code may have established a stress state (p, q, θ) and a void ratio e, but the stored hardening variable may not be consistent with the current stress point and the CASM yield surface.

The UMAT's `setCustomVariable()` performs a **post-equilibrium conditioning** procedure that mirrors the "a_0, b_0" idea described in the GCC documentation (see gccmodel.md), but without any unsaturation/hydraulic coupling. This ensures that after initialization, the stress state lies either on or inside the yield surface.

1.9.1 Step 1 — Fit the minimum cap size through the current stress

From the current stress invariants, compute

- $p = -\sigma_m$ (compression-positive)
- q (equivalent deviatoric stress)
- F_θ (Lode-angle factor)

Then compute the **minimum** cap size $p_{0,\min}$ that makes the yield function pass exactly through the current stress state (i.e., $f = 0$):

$$p_{0,\min} = \frac{p}{\exp\left(-\frac{1}{r^*}\left(\frac{qF_\theta}{Mp}\right)^n\right)} \tag{11}$$

In the code, p is locally bounded using `P_min` (only for log/division safety) to avoid numerical issues if the mean stress is extremely small. This local bound does **not** affect the deviatoric stress invariants.

1.9.2 Step 2 — Apply a_0 and b_0 (via `DefaultIsoHardening` and `OCR`)

The UMAT then initializes the stored hardening variable (`IsotropicHardening`) as:

$$p_{0,\text{init}} = \max\left(p_{0,\min},\ a_0 + b_0\, p_{0,\min}\right) \tag{12}$$

with

- $a_0 =$ `DefaultIsoHardening`
- $b_0 =$ `OCR` (optional; defaults to 0)

So:

- If you set `OCR = 0`, then $p_{0,\text{init}} = \max(p_{0,\min}, a_0)$.
- Larger `OCR` increases the initialized cap size proportionally to $p_{0,\min}$, pushing the stress point further inside the yield surface (higher apparent overconsolidation).

1.9.3 Step 3 — Update void ratio to be consistent with the initialized cap

Finally, the void ratio is updated using the elastic unloading branch of the NCL:

$$v = v_N + \kappa \left(\ln(p_{0,\text{init}}) - \ln(p)\right) - \lambda \ln(p_{0,\text{init}}), \qquad e = v - 1 \tag{13}$$

This ensures that (p, e, p_0) are mutually consistent with the elastoplastic framework at the start of loading.

1.10 Numerical integration

The UMAT uses **adaptive substepping** with error control governed by `STOL`. Each substep applies:

1. **Elastic trial** — compute trial stress assuming purely elastic response.
2. **Yield check** — evaluate f at the trial stress; if $f \leq$ FTOL, accept elastic.
3. **Load–unload detection** — use `LTOL` and a regula-falsi (Illinois) intersection finder to locate the yield-surface crossing within the substep.
4. **Plastic return** — enforce consistency via the implicit return-mapping algorithm.
5. **Drift correction** — iteratively project the stress back onto the yield surface if numerical drift exceeds `FTOL`.

Denominator guards and numerical safeguards are applied throughout to prevent division-by-zero and NaN propagation.

1.11 Example input (triaxial driver)

```
# --- CASM driver input (units: stress in kPa; compression negative) ---
Mode  Drained

nSteps     2000
dEpsAxial  -1e-4

# Material parameters
Phi   23
Lambda 0.093
Nu     0.30
Kappa  0.025
Alpha  0.78
SSC    4.5
SPR    2.714
P_min  0.1
DefaultIsoHardening 207.5
v_N    2.1071
```

```
# Optional: post-equilibrium multiplier
# OCR 0.0

# Solver tolerances
STOL 1e-7
FTOL 1e-4
LTOL 1e-6

# Initial state
VoidRatio 0.632
StressXX  -207
StressYY  -207
StressZZ  -207

# Custom state (if omitted, post-equilibrium conditioning will set it)
# IsotropicHardening 207.5
```

1.12 Verification: Drained Triaxial Tests on Weald Clay

To verify the CASM implementation, drained triaxial compression tests on Weald Clay were simulated under two different overconsolidation conditions. The model shows generally good agreement with experimental data across both normally consolidated and heavily overconsolidated states.

1.12.1 Test Conditions and Parameters

Material Parameters (Weald Clay)

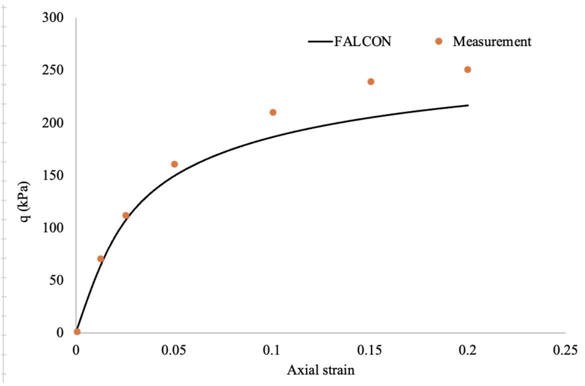
Symbol	Keyword	Value
ϕ	`Phi`	23.0°
λ	`Lambda`	0.093
κ	`Kappa`	0.025
ν	`Nu`	0.30
v_N	`v_N`	2.1071
α	`Alpha`	0.78
n	`SSC`	4.5
r	`SPR`	2.714

Initial Conditions

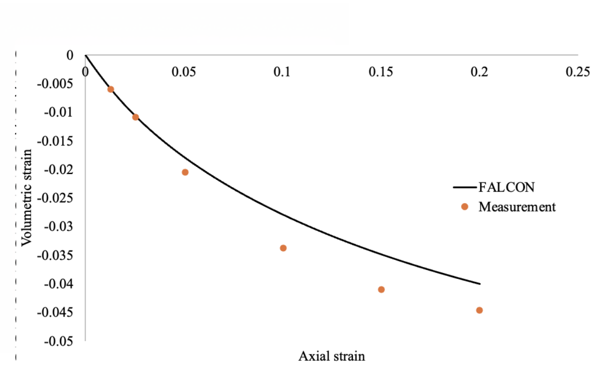
Test	OCR	e_0	p_0 (kPa)
Test 1: Normally Consolidated	1	0.632	207
Test 2: Heavily Overconsolidated	24	0.617	34.5

1.12.2 Results

Normally Consolidated (OCR = 1)

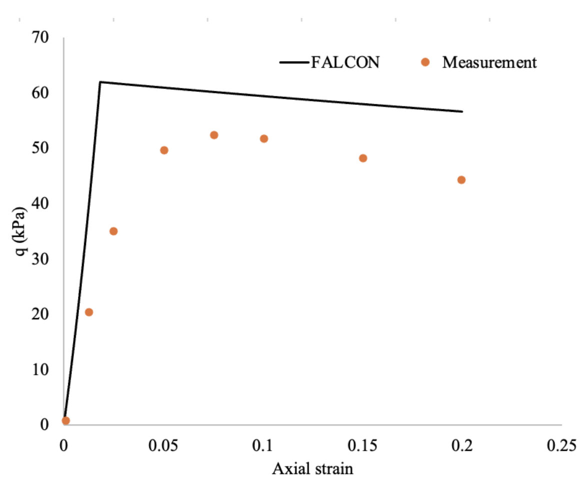


(a) Deviatoric stress vs axial strain

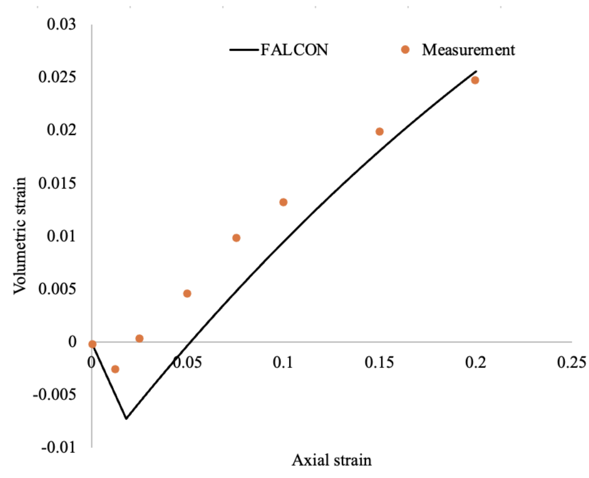


(b) Volumetric strain vs axial strain

Overconsolidated (OCR = 24)



(c) Deviatoric stress vs axial strain



(d) Volumetric strain vs axial strain

The simulations demonstrate the model's capability to capture:

- Peak strength and post-peak softening behavior
- Contractive response for normally consolidated clay
- Dilative response for heavily overconsolidated clay
- Transition from elastic to plastic deformation

1.13 References

- Yu, H. S. (1998). *CASM: a unified state parameter model for clay and sand.* International Journal for Numerical and Analytical Methods in Geomechanics, 22(8), 621–653.