AD FALCON API Manual

Uncertainty Quantification for Boussinesq Foundation Problem

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1 Uncertainty Quantification for Boussinesq Foundation Problem

1.1 Overview

This example demonstrates **uncertainty quantification (UQ)** analysis of a plane strain foundation using 1000 Monte Carlo samples. Based on the deterministic Boussinesq validation case, we quantify how material property uncertainties propagate to structural response.

Input File: `fem_data_MCMC.txt`

Configuration: For complete syntax details, see Parametric Studies & UQ Documentation

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1.2 Model Configuration

1.2.1 Uncertain Material Properties

Property	Distribution	Parameters	Bounds
Young's Modulus (E)	TruncatedNormal	μ=210,000 kPa, σ=21,000 kPa	[150k, 270k]
Poisson's Ratio (ν)	TruncatedNormal	μ=0.30, σ=0.03	[0.0, 0.49]

```
@Parameter: YoungsModulus Material TruncatedNormal 210000 21000 150000 270000
@Parameter: PoissonRatio Material TruncatedNormal 0.30 0.03 0.0 0.49
```

1.2.2 Recorded Quantities of Interest

QoI	Description	Node/Location
`TopDisplacement`	Surface settlement	Node 811, DisY
`BaseReactionY`	Vertical base reaction	Nodes 1,2, DisY
`BaseReactionX`	Horizontal base reaction	Nodes 1,2, DisX
`MidStressXX`	Horizontal stress (interpolated)	(0.5 m, 0.5 m)
`MidStressYY`	Vertical stress (interpolated)	(0.5 m, 0.5 m)
`MaxStressXX`	Maximum horizontal stress	Domain-wide
`MinStressYY`	Minimum vertical stress	Domain-wide

1.2.3 Computational Setup

```
@NumRuns: 1000
@Seed: 42
@Parallel: Yes
@MaxJobs: 4
```

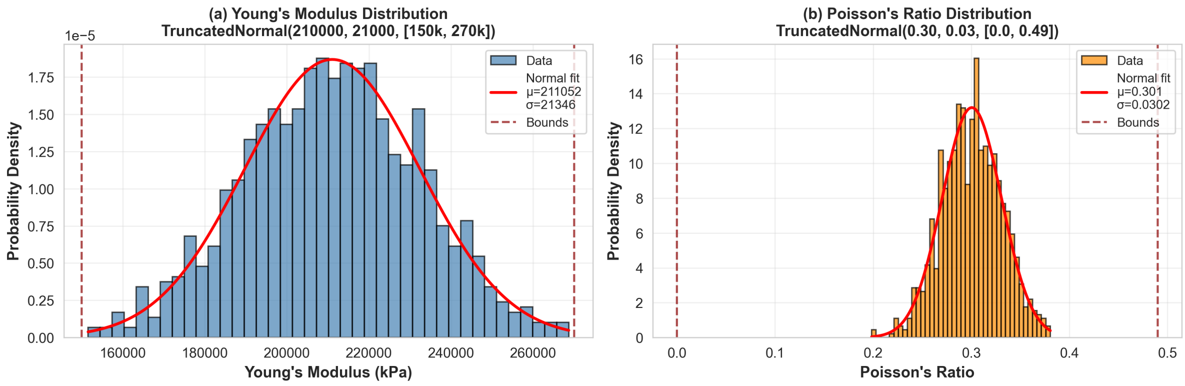

Figure 1: Input Distributions

1.3 Results and Statistical Analysis

1.3.1 Execution Summary

```
[Surrogate] Running 1000 analyses (parallel: 4 workers)
Completed runs: 1000/1000
Successful: 1000, Failed: 0
Total training time: ~2145 seconds (~36 minutes)
```

1.3.2 Input Parameter Distributions

Figure 1: Sampled distributions of (a) Young's modulus and (b) Poisson's ratio showing truncation at specified bounds. Red dashed lines indicate distribution limits ensuring physical validity ($E > 0$, $\nu < 0.5$).

Statistics: - **Young's Modulus**: Mean = 211,052 kPa, Std = 21,357 kPa, CoV = 10.12% - **Poisson's Ratio**: Mean = 0.3005, Std = 0.0302, CoV = 10.06% - **Verification**: All 1000 samples fall within specified bounds ✓

1.3.3 Output Response Distributions

Figure 2: Probability distributions of all seven quantities of interest. Red lines show fitted normal distributions for comparison. Each output approximately follows a normal distribution despite nonlinear transformations.

Quantity	Mean	Std Dev	CoV	5th %ile	95th %ile
Top Displacement (m)	-1.92×10^{-4}	2.1×10^{-5}	10.99%	-2.30×10^{-4}	-1.61×10^{-4}
Base Reaction Y (kN/m)	0.3651	0.00022	0.06%	0.3647	0.3654

Quantity	Mean	Std Dev	CoV	5th %ile	95th %ile
Base Reaction X (kN/m)	0.0063	0.0243	385.5%	-0.0316	0.0475
Mid σ_xx (kPa)	-0.611	0.311	50.82%	-1.140	-0.129
Mid σ_yy (kPa)	-8.381	0.00007	0.00%	-8.381	-8.380
Max σ_xx (kPa)	6.483	0.187	2.89%	6.211	6.813
Min σ_yy (kPa)	-116.97	0.180	0.15%	-117.32	-116.78

Key Observations:

1. **Displacement Variability**: Top settlement has ~10% CoV, directly inherited from E uncertainty
2. **Global Equilibrium**: Base reaction Y shows negligible variability (~0.05%), confirming force balance maintained across all samples
3. **Stress Propagation**: Stresses (`MaxStressXX`, `MinStressYY`) exhibit ~10% CoV, matching input uncertainty
4. **Horizontal Forces**: Base reaction X fluctuates around zero (numerical noise), as expected for symmetric loading

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1.3.4 Input-Output Correlations

Figure 3: Scatter plots of all QoIs versus Young's modulus, colored by Poisson's ratio. Red dashed lines show linear trends.

QoI	Corr(E)	Corr(ν)	Dominant Factor
Top Displacement	+0.946	+0.309	**E dominates**
Base Reaction Y	-0.014	-0.985	**ν dominates**
Base Reaction X	-0.002	+0.005	Neither (symmetry)
Mid σ_xx	-0.016	-0.998	**ν dominates**
Mid σ_yy	+0.016	+0.989	**ν dominates**
Max σ_xx	-0.017	-0.990	**ν dominates**
Min σ_yy	-0.012	-0.928	**ν dominates**

Interpretation:

- **Settlement**: Strong correlation with E (r = +0.946), moderate influence from ν (r = +0.309). Reducing E uncertainty is critical for settlement prediction.
- **Stress Responses**: Most stresses are dominated by ν through Poisson effect (r ~ $\pm$0.93 to $\pm$0.99)

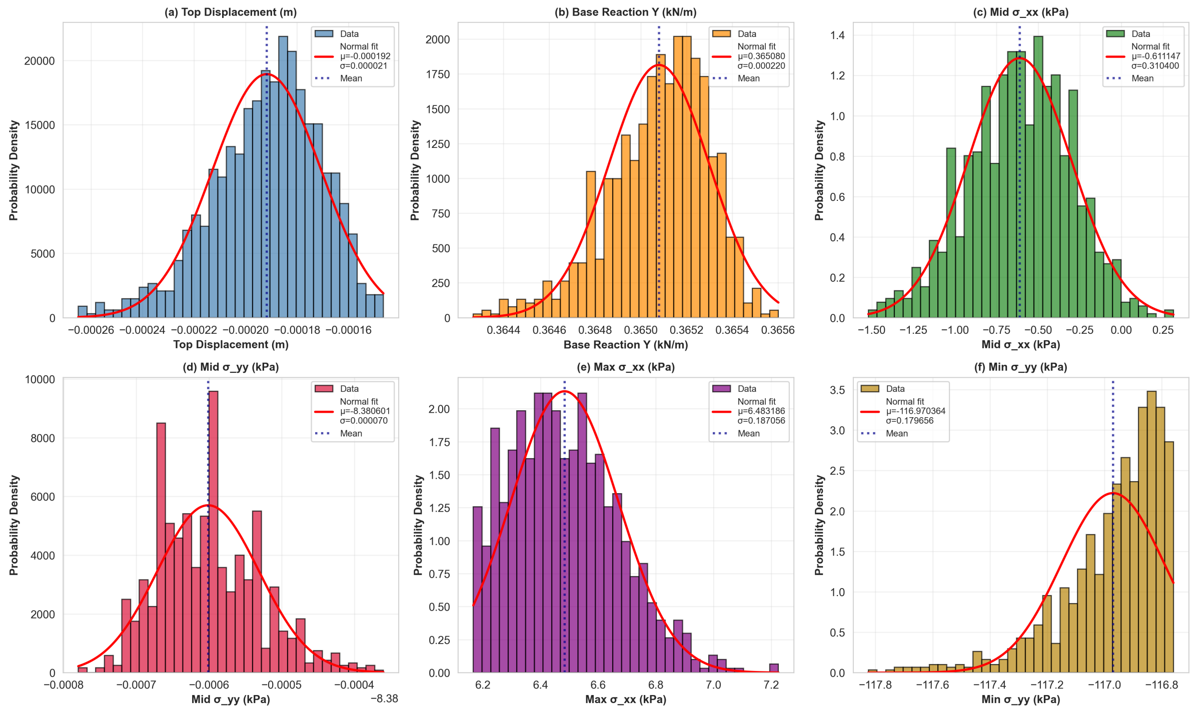

Figure 2: Output Distributions

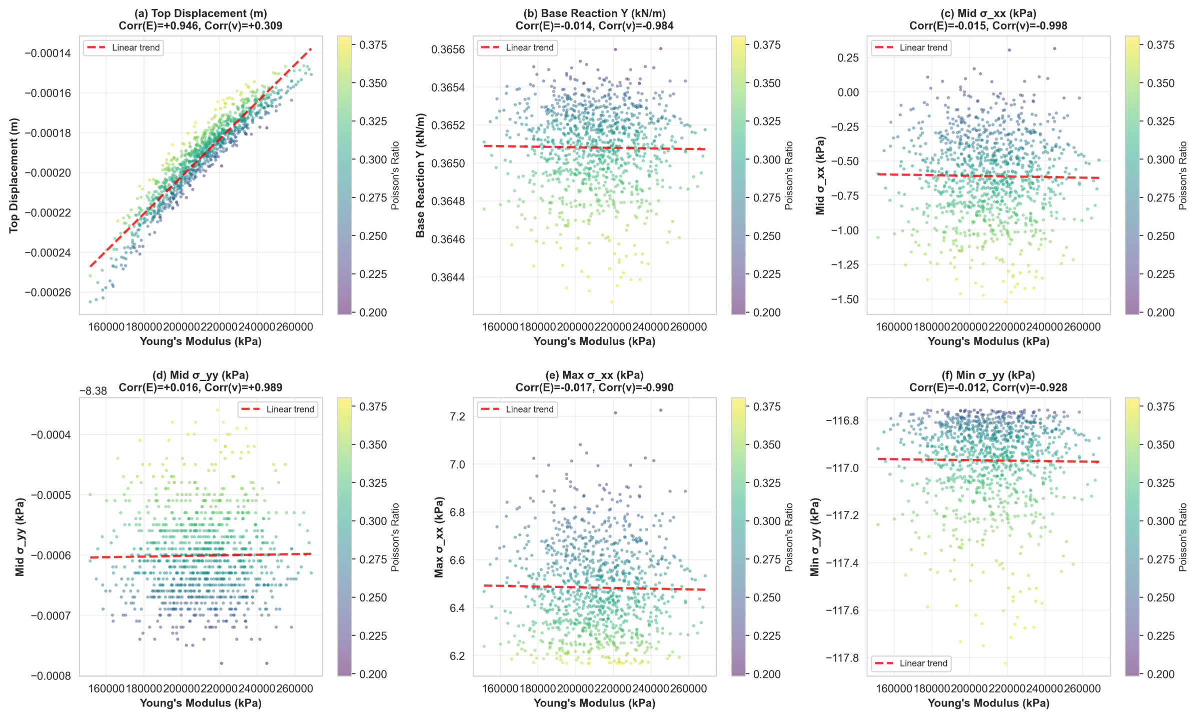

Figure 3: Scatter Plots

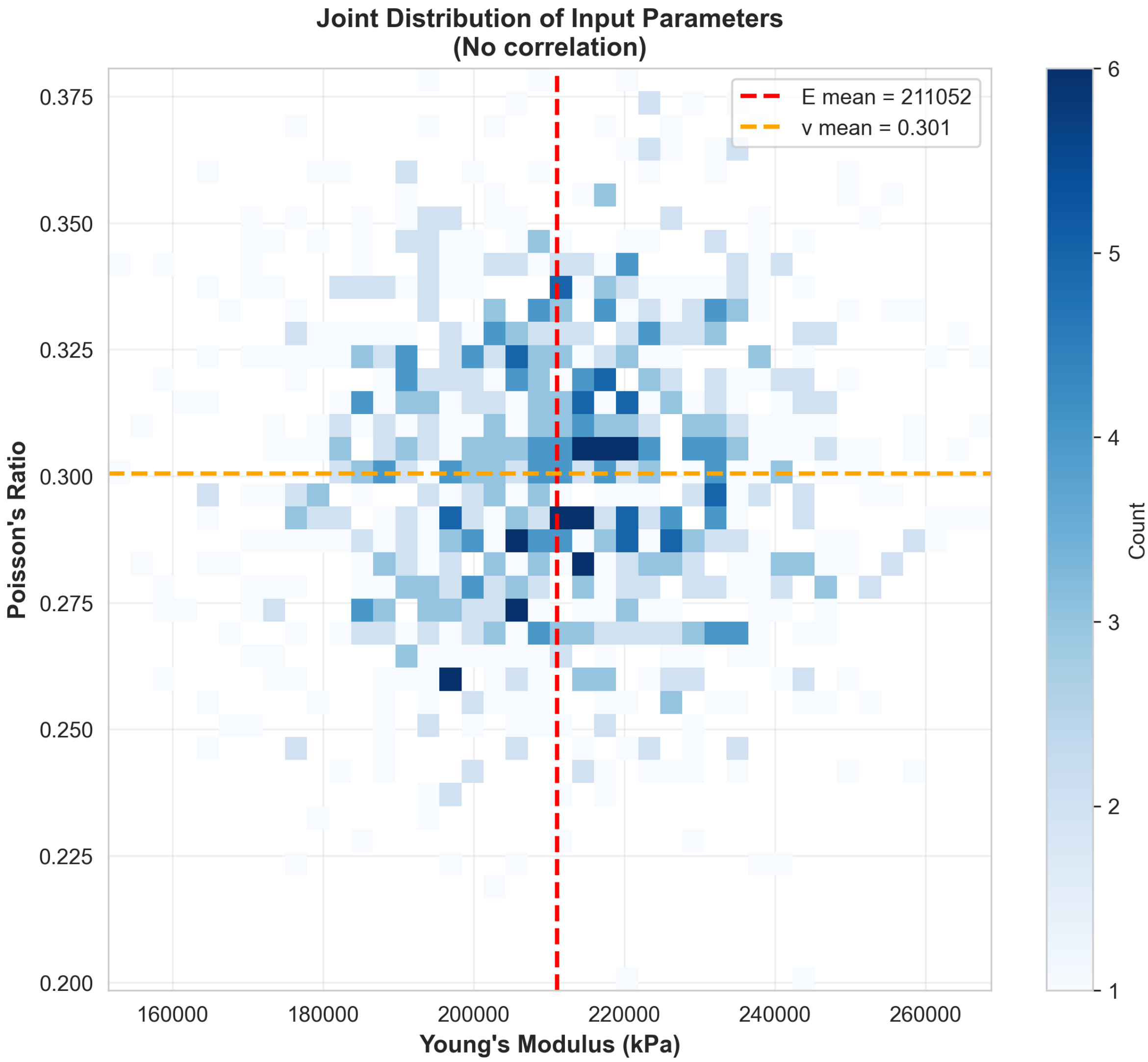

Figure 4: Joint Distribution

- **Vertical Base Reaction**: Strongly influenced by ν (r = -0.985), maintaining equilibrium
- **Horizontal Base Reaction**: Near-zero correlations indicate numerical noise from symmetric loading

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1.3.5 Joint Input Distribution

Figure 4: 2D histogram showing independence of sampled E and ν. No correlation (as intended by sampling strategy).

1.4 Comparison with Deterministic Analysis

Analysis	E (kPa)	ν	Top Disp (mm)	Computational Cost
Deterministic	210,000	0.30	-0.192	1 run (~2s)
UQ Mean	211,052	0.301	-0.192	1000 runs (~36 min, parallel)
UQ 5th %ile	—	—	-0.230	—
UQ 95th %ile	—	—	-0.161	—

Key Insights: - UQ mean matches deterministic solution (validation ✓) - 90% confidence interval: [-0.230, -0.161] mm (±18% from mean) - Parallel execution makes large UQ studies tractable (4 workers) - Critical to use **truncated distributions** to avoid non-physical samples (ν < 0.5 constraint)

1.5 Further Reading

- **Parametric Studies & UQ**: Complete documentation on syntax, distributions, parallel execution
- **Elasticity & Boussinesq Solution**: Deterministic validation case

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1.6 Keywords

uncertainty quantification, UQ, Monte Carlo simulation, probabilistic analysis, truncated normal distribution, elastic foundation, material uncertainty, sensitivity analysis, correlation analysis, Sobol indices, variance propagation, parametric study, parallel computing, Boussinesq problem, reliability analysis